Friday

1. U-sub basics
2. Not so u-sub basics

warm-up

$$\int_{x=1}^{x=3} x(x^2+1)\,dx = ?$$

$$\boxed{u = x^2+1} \longrightarrow \begin{array}{l} x=1 \Rightarrow u = 1^2+1 = 2 \\ x=3 \Rightarrow u = 10 \end{array}$$

$$\frac{d}{dx}(u) = \frac{d}{dx}(x^2+1)$$

$$\frac{du}{dx} = 2x$$

$$du = 2x\,dx$$

$$\boxed{\frac{1}{2x}du = dx}$$

$$= \int_{\square}^{\square} x \cdot u \, \frac{1}{2x} du$$

$$= \frac{1}{2}\int_2^{10} u\,du$$

$$= \frac{1}{2}\left(\frac{u^2}{2}\right)\Big|_2^{10} = \frac{1}{2}\left[\frac{100}{2} - \frac{4}{2}\right] = \frac{1}{2}[50-2] = \frac{1}{2}[48] = 24$$

Fri - Wk 1 ———— Substitution (easy $\longrightarrow$ less so) 10:13 start

① $\int e^{7x+1}\,dx = \int e^{u}\,\frac{1}{7}du = \frac{1}{7}\int e^{u}du = \boxed{\frac{1}{7}e^{7x+1}+C}$

recall! $\int e^{u}du = e^{u}+C$ $\longleftarrow$

$\boxed{u = 7x+1}$ $\boxed{\frac{1}{7}du = dx}$

$\frac{du}{dx} = 7$

$du = 7dx$

② $\int 3x\cdot\cos(x^2)\,dx = 3\int x\cos(u)\,dx$

$u = x^2$

$du = 2x\,dx$

$= \frac{3}{2}\int \cos(u)\,2x\,dx$

$= \frac{3}{2}\int \cos(u)\,du$

$= \boxed{\frac{3}{2}\sin(x^2)+C}$

$$\boxed{\begin{aligned}\frac{d}{dx}(\text{ans}) &= \frac{3}{2}\cos(x^2)\cdot 2x\\ &= 3\cos(x^2)\cdot x\end{aligned}}$$

③ $\int \frac{e^{\ln(x)}}{x}\,dx = \int e^{\ln x}\boxed{\frac{1}{x}\,dx}$

$= \int e^{u}du = \boxed{e^{\ln x}+C}$

$u = \ln(x)$

$du = \boxed{\frac{1}{x}dx}$

check by differentiating your answer.

④ $\int \frac{2x}{(5x^2+1)^3}\,dx = \frac{1}{5}\int \frac{5\cdot 2x}{u^3}\,dx$

$u = 5x^2+1$

$du = 10x\,dx$

$= \frac{1}{5}\int \frac{10x\,dx}{u^3}$

$= \frac{1}{5}\int \frac{du}{u^3}$

$= \frac{1}{5}\int u^{-3}du =$

$= \frac{1}{5}\left(\frac{u^{-2}}{-2}\right)+C$

$= \frac{-u^{-2}}{10}+C$

$= -\frac{(5x^2+1)^{-2}}{10}+C$

$= \frac{-1}{10(5x^2+1)^2}+C$

check:

$\frac{d}{dx}(\text{ans}\#4) = \frac{(\cancel{-}2)(\cancel{-}(5x^2+1)^{-3}\cdot(\cancel{10}x)}{\cancel{10}} = \frac{2x}{(5x^2+1)^3}$ ✓

Less Basic Substitution

① $\int \frac{x}{\sqrt{x+1}}\,dx \longrightarrow \int \frac{u-1}{\sqrt{u}}\,du = \int \frac{u}{\sqrt{u}} - \frac{1}{\sqrt{u}}\,du$

$\boxed{u = x+1} \xrightarrow{-1} \boxed{u-1 = x}$

$\frac{u^1}{u^{1/2}} = u^1 \cdot u^{-1/2} = u^{1/2}$

$du = dx$

$$= \int \frac{x}{\sqrt{u}}\,du = \int \frac{u-1}{\sqrt{u}} = \int \frac{u}{\sqrt{u}} - \frac{1}{\sqrt{u}}\,du = \int u^{1/2} - u^{-1/2}\,du$$

$$= \frac{2}{3}u^{\frac{3}{2}} - \frac{2}{1}u^{\frac{1}{2}} + C = \boxed{\frac{2}{3}(x+1)^{\frac{3}{2}} - 2(x+1)^{\frac{1}{2}} + C}$$

② $\int \frac{x^3}{\sqrt{x^2-1}}\,dx = \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}} \cdot x^2\,dx = \frac{1}{2}\int \frac{1}{u^{1/2}}(u+1)\,du$

$\boxed{u = x^2-1} \longrightarrow \boxed{u+1 = x^2}$

$\boxed{du = 2x\,dx}$

$$= \frac{1}{2}\int \frac{u+1}{u^{1/2}}\,du = \frac{1}{2}\int \frac{u}{u^{1/2}} + \frac{1}{u^{1/2}}\,du$$

$$= \frac{1}{2}\int u^{1/2} + u^{-1/2}\,du = \frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}} + \frac{2}{1}u^{\frac{1}{2}}\right] + C$$

$$= \frac{1}{2}\left[\frac{2}{3}(x^2-1)^{\frac{3}{2}} + 2(x^2-1)^{\frac{1}{2}}\right] + C$$

(3) $\int \sin^4 x \cdot \cos x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$\downarrow$

$\int u^4 \, du = \frac{u^5}{5} + C = \boxed{\frac{\sin^5 x}{5} + C}$

$\int \tan x \sec x \cdot \sec x$

$\tan^2 x = \sec^2 x + 1$

(4) $\int \tan x \cdot \sec^2 x \, dx$

$u = \tan x$

$du = \sec^2 x$

$\int u \, du = \frac{u^2}{2} + c$

$\boxed{\frac{\tan^2 x}{2} + C}$

or

$u = \sec x$

$du = \sec x \tan x$

$\int u \, du = \frac{u^2}{2} + c$

$\boxed{\frac{\sec^2 x}{2} + C}$

Substitution!

(warm-up)

$$\int_{x=1}^{x=3} x(x^2+1)^3\,dx =$$

$\boxed{u = x^2+1}$ $\quad x=1 \Rightarrow u = 1^2+1 = 2$

$x = 3 \Rightarrow u = 10$

$$\frac{d}{dx}(u) = \frac{d}{dx}(x^2+1)$$

$$\| $$

$$\frac{du}{dx} = 2x \quad \text{so} \quad du = 2x\,dx$$

↑

treat like fraction

$$\boxed{\frac{1}{2x}du = dx}$$

seating chart (Friday wk 1)

Bernie 

Brenan

Shane

Nicholas

Alex Tyler

Ethan

Josh Hudson McKenzie

Jesse

Fact: $\int dx$ is linear //

$$= \int x(u)^3 \frac{1}{2x}\,du = \frac{1}{2}\int \cancel{x}\cdot u^3 \frac{1}{\cancel{x}}\,du$$

$$(\star) = \frac{1}{2}\int_2^{10} u^3\,du = \frac{1}{2}\frac{u^4}{4}$$

$$= \frac{u^4}{8}\Big|_2^{10} = \frac{10^4}{8} - \frac{2^4}{8}$$

$$= \frac{(9984)}{8}$$

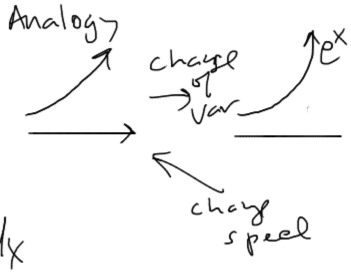
$$= \frac{(x^2+1)^4}{8}\Big|_1^3$$

same

Fnd Wk 1

substitution : easy $\longrightarrow$ less so

Analogy



① $\frac{1}{7}\int e^{7x+1}\, 7dx$

$u = 7x+1$

$du = 7dx$

Know: $\int e^u du = e^u$

$$= \frac{1}{7}\int e^u du = \frac{1}{7}e^u + c = \boxed{\frac{1}{7}e^{7x+1} + c}$$

③ $\int \frac{\ln x}{x} dx$

$u = \ln x$

$du = \frac{1}{x}dx$

$$= \int \ln x \cdot \frac{1}{x} dx$$

$$= \int u \cdot du = \frac{u^2}{2} + c = \boxed{\frac{(\ln x)^2}{2} + c}$$

② $\int 3x \cdot \cos(x^2)\, dx \Leftrightarrow k\int \cos(u)\, du$

$\boxed{u = x^2}$

$du = 2x\,dx$

$\boxed{\frac{1}{2x}du = dx}$

$$= 3\int \cancel{x} \cdot \cos(u) \frac{1}{2\cancel{x}} du$$

$$= \frac{3}{2}\int \cos(u)\, du$$

$$= \frac{3}{2}\sin(u) + c$$

$$= \boxed{\frac{3}{2}\sin(x^2) + C}$$

④ $\frac{1}{5}\int \frac{5 \cdot 2x}{(5x^2+1)^3} dx = \frac{1}{5}\int \frac{du}{u^3}$

$u = 5x^2 + 1$

$du = 10x\, dx$

$$= \frac{1}{5}\int u^{-3} du$$

$$= \frac{1}{5}\left(\frac{u^{-2}}{-2}\right) + C$$

$$\int \frac{1}{x^3} dx = \int x^{-3} dx = \frac{x^{-2}}{-2} + C$$

$$= \boxed{\frac{(5x^2+1)^{-2}}{-10} + c}$$

Less Basic u-sub

$\frac{4-1}{3} = \frac{4}{3} - \frac{1}{3}$

① $\int \frac{x}{\sqrt{x+1}}\,dx = \int \frac{u-1}{\sqrt{u}}\,du = \int \frac{u}{\sqrt{u}} - \frac{1}{\sqrt{u}}\,du$

$u = x+1 \longrightarrow u-1 = x$

$du = dx$

$= \int u^{1/2} - u^{-1/2}\,du$

$\frac{u}{u^{1/2}} = u^1 \cdot u^{-1/2}$

$= u^{1/2}$

$\frac{2}{3}u^{3/2} - 2u^{1/2} + c$

$\boxed{\frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2} + C}$

② $\int \frac{x^3}{\sqrt{x^2-1}}\,dx$

try: $u = x^2 - 1$

$\frac{1}{2}\ x^3 = x \cdot (x^2)$