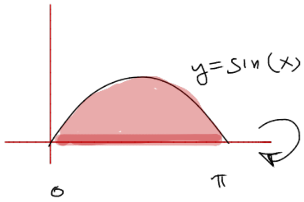
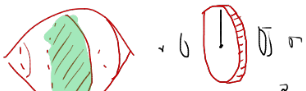
Wk 2 — Fri

Revolve region about x-axis, Compute Volume





$V = \pi r^2 \cdot \Delta x$

$= \int_0^{\pi} \pi r^2 \, dx = \int_0^{\pi} \pi \sin^2 x \, dx$

r = radius = sin x

$\frac{1}{2}\sin 2x$

$\uparrow \int dx$

$\cos 2x$

$\frac{d}{dx} \downarrow$

$-\sin 2x \cdot 2$

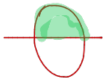
$$= \pi \int_0^{\pi} \frac{1 - \cos(2x)}{2} \, dx = \frac{\pi}{2} \int_0^{\pi} 1 - \cos(2x) \, dx = \frac{\pi}{2} \left[x - \frac{1}{2} \sin(2x) \right]_0^{\pi}$$

$$= \frac{\pi}{2} \left[\pi - \frac{\overset{0}{\sin(2\pi)}}{2} - \left(0 - \frac{\overset{0}{\sin(0)}}{2} \right) \right]$$

$= \frac{\pi^2}{2}$ cubic units

Today: $\int \sec^m(x) \tan^n(x) \, dx$ + Trig Substitution (7-3)

↳ $\int \sqrt{1 - x^2} \, dx$



$$\int \sec^{m}(x)\cdot\tan^{n}(x)\,dx$$

if

- $\tan(x)$ power $= n =$ odd $\Longrightarrow$ strip one $\tan(x)$ and one $\sec(x)$ $\Big\}$ $\sec(x)\tan(x) \overset{\text{want}}{=} du$ $\boxed{\text{set: } u=\sec(x)}$

  use pythagorean!

- $\sec(x)$ power $= m =$ even $\Rightarrow$ strip $\sec^2(x)$ $\Rightarrow$ $\sec^2(x) \overset{\text{want}}{=} du$

  set: $u=\tan(x)$

- neither? $\Rightarrow$ use pythagorean $\Rightarrow$ $\int \sec^{\text{odd}}(x)\,dx =$ table or similar $\int \sec^3(x)\,dx$

Ex

① $$\int \sec^3(x)\tan^3(x)\,dx = \int \sec^2(x)\cdot\tan^2(x)\cdot\underbrace{\sec(x)\tan(x)\,dx}_{du}$$

get in terms of $\sec(x)$

$u = \sec(x)$

$$= \int \sec^2(x)\cdot(\sec^2(x)-1)\,\underbrace{\sec(x)\tan(x)\,dx} = \int u^2(u^2-1)\,du$$

$u = \sec(x)$

$du = \sec(x)\tan(x)\,dx$

$$= \int u^4 - u^2\,du$$

$$= \frac{u^5}{5} - \frac{u^3}{3} + C$$

$$= \boxed{\frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3} + C}$$

② $$\int \sec^4(x)\cdot\tan^2(x)\,dx = \int \sec^2(x)\cdot\tan^2(x)\cdot\underbrace{\sec^2(x)\,dx}_{du}$$

even $\sec(x)$ power

$\sec^2(x) \downarrow (\tan^2(x)+1)$

$u = \tan(x)$

$$= \int (u^2+1)u^2\,du = \int u^4 + u^2\,du = \boxed{\frac{\tan^5(x)}{5} + \frac{\tan^3(x)}{3} + C}$$

③ $$\overset{=A}{\int \sec^3(x)\tan^2(x)\,dx} \overset{\text{table?}}{=}$$

pythag $\Rightarrow \int \sec^{\text{odd}}$

$$= \int \sec^3(x)(\sec^2(x)-1)\,dx = \int \sec^5(x) - \sec^3(x)\,dx$$

$$\boxed{\begin{aligned} A &= \int \sec^5 - \int \sec^3 \\ &\quad \sec^3\tan(x) - 3A \quad\quad D \\ A &= \frac{\sec^3\tan x - D}{4}\end{aligned}}$$

odd $\sec(x)$ technique

$$\int \sec^5(x)\,dx = \boxed{\sec^3(x)\tan(x)} - 3\int \sec^3(x)\tan^2(x)\,dx$$

$u = \sec^3(x)$ $\quad dv = \sec^2(x)\,dx$

$du = 3\sec^3(x)\tan(x)$ $\quad v = \tan(x)$

$$\int \sec^3(x)\tan^2(x) =$$

$$\frac{1}{4}\left[\sec^3\tan(x) + \frac{1}{2}\left[\sec(x)\tan(x) - \ln|\sec(x)+\tan(x)|\right]\right]$$

$$\int \sec^3(x)\,dx = \text{I.B.P.}$$

$u = \sec(x)$ $\quad dv = \sec^2(x)$

$du = \sec(x)\tan(x)$ $\quad v = \tan(x)$

$$= \sec(x)\tan(x) - \int \sec(x)\tan^2(x)\,dx$$

$$\tan^2(x) = (\sec^2(x)-1)$$

$$-\int \sec^3(x) - \sec(x)\,dx$$

$$-\int \sec^3(x) - \int \sec(x)\,dx$$

$$\int \sec(x)\,dx = -\ln|\sec(x)+\tan(x)|$$

$$\int \sec^3(x)\,dx = \frac{1}{2}\left[\sec(x)\tan(x) - \ln|\sec(x)+\tan(x)|\right]$$

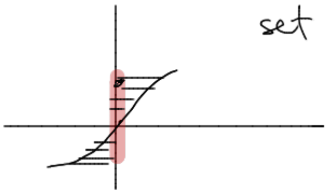
$= D$

TRIG SUB

$$\int \sqrt{1 - x^2}\, dx = \int \cos\theta \cos\theta \, d\theta = \int \cos^2\theta \, d\theta$$

$\underline{x \in [-1, 1]}$

$$\sqrt{1 - \sin^2\theta} = \sqrt{\cos^2\theta} = \underline{\cos\theta}$$



set $x = \sin\theta$

$dx = \cos\theta$