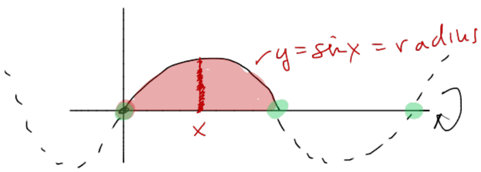
Wk 2 – Fri

Today: sec(x)tan(x) integrals + trig substitution

Warm-up: compute vol. of solid of revol.

$y = \sin(x)$ revolve about the x-axis, w/ bounds $x=0$, $x=\pi$



vol. of slice = surf. area × thickness

$= \pi r^2 \cdot \Delta x = \pi \sin^2 x \,\Delta x$

$$V = \int_0^{\pi} \pi \sin^2 x \, dx = \pi \int_0^{\pi} \sin^2 x \, dx$$

$\frac{1}{2}\int \cos(2x)\, 2\, dx = \frac{1}{2}\int \cos(u)\, du$

$$= \pi \int_0^{\pi} \frac{1-\cos(2x)}{2}\, dx = \frac{\pi}{2}\int_0^{\pi} 1 - \cos(2x)\, dx = \frac{\pi}{2}\left[x - \frac{1}{2}\sin(2x)\right]\Big|_0^{\pi}$$

$$= \frac{\pi}{2}\left[\pi - \frac{1}{2}\underbrace{\sin(2\pi)}_{=0} - \left(0 - \frac{1}{2}\underbrace{\sin(0)}_{=0}\right)\right] = \frac{\pi^2}{2} \text{ cubic units}$$

$$\int \sec^m(x)\tan^n(x)\,dx$$

if:

- $\sec(x)$ power $= m$ (even) $\Rightarrow$ strip off $\sec^2(x)$, which becomes du
  $u = \tan(x)$
  (use pythagorean)

- $\tan(x)$ power $= n$ (odd) $\Rightarrow$ strip off $\tan(x)$ AND $\sec(x)$
  so $\sec(x)\tan(x)$ becomes du
  thus $u = \sec(x)$

- neither? use pythagorean $\Rightarrow$ $\int \sec^{\text{odd}}(x)\,dx$ (similar to $\int \sec^3(x)\,dx$)

Ex

① $\int \sec^2(x)\tan^2(x)\,dx = \int u^2\,du = \frac{u^3}{3} + c = \frac{\tan^3(x)}{3} + c$

$u = \tan(x)$

② $\int \sec^4(x)\tan^2(x)\,dx = \int \underbrace{\sec^2(x)}_{=(1+\tan^2(x))} \tan^2(x) \cdot \sec^2(x)\,dx$

think: $u = \tan(x)$ $du = \sec^2(x)\,dx$

issue: get the remaining $\sec(x)$ in terms of $\tan(x)$

$= \int (1+\tan^2(x))\tan^2(x) \cdot \sec^2(x)\,dx$

distribute NOW

u-sub early

$\int (1+u^2)u^2\,du = \int u^2 + u^4\,du = \frac{u^3}{3} + \frac{u^5}{5} + c$

$= \boxed{\frac{\tan^3(x)}{3} + \frac{\tan^5(x)}{5} + C}$

$= \int (\tan^2(x) + \tan^4(x)) \sec^2(x)\,dx$

$u = \tan$

$du = \sec^2$

$= \int (u^2 + u^4)\,du$

TIPS TO Remember

odd vs. even

$\tan(x)$, $\sin(x)$, $\cos(x)$ $\sec(x)$

$$\int \sec^5(x)\tan^3(x)\,dx = \int \sec^4(x)\underbrace{\tan^2(x)}_{=\sec^2(x)-1}\cdot\sec(x)\tan(x)\,dx$$

Notice: odd tan power

want: du

set $u = \sec(x)$

$du = \sec(x)\tan(x)\,dx$

pythag

$$= \int \sec^4(x)(\sec^2(x)-1)\sec(x)\tan(x)\,dx$$

$$= \int u^4(u^2-1)\,du = \int u^6 - u^4\,du = \frac{u^7}{7} - \frac{u^5}{5} + C$$

$$= \boxed{\frac{\sec^7(x)}{7} - \frac{\sec^5(x)}{5} + C}$$

Ex.

$$\int \sec^3(x)\tan^2(x)\,dx$$

bummer, sec is odd
tan is even

$$= \int \sec^3(x)\left(\sec^2(x) - 1\right) dx$$

$$= \int \sec^5(x) - \sec^3(x)\,dx$$

= see table !

$$\tan^2 x = \sec^2 x - 1$$

$$\tan^4 x = (\tan^2 x)^2$$

$$= (\sec^2 x - 1)^2$$

$$= 2\sec^4 x - 2\sec^2 x + 1$$

$$\tan^6 x = (\tan^2 x)^3 = (\sec^2 - 1)^3$$

$$= 3\cdot\sec^6 x + 3\sec^2 x + 3 + 1$$

TRIG SUBSTITUTION

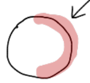
Integrals like $\int \frac{1}{\sqrt{9-x^2}}\,dx$, $\int \sqrt{4-x^2}\,dx$, ...

$$\boxed{\int \frac{1}{\sqrt{9-x^2}}\,dx}$$

idea: exploit the domain to get a proper substitution

$\sqrt{9-x^2}$ what values can x have ?

$\cos\theta > 0$



$9 - x^2 \le 0$ $\quad$ $9 \le x^2$

$\Rightarrow \boxed{-3 \le x \le 3}$.

set $\boxed{x = 3\sin\theta, \ \text{w/}\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}}$

$dx = 3\cos\theta\, d\theta$

$$\sqrt{9-x^2} = \sqrt{9-(3\sin\theta)^2} = \sqrt{9 - 9\sin^2\theta} = \sqrt{9(1-\sin^2\theta)} = 3\sqrt{(1-\sin^2\theta)} = 3\sqrt{\cos^2\theta} = 3|\cos\theta| = 3\cos\theta$$

$$= \int \frac{1}{3\cos\theta}\, 3\cos\theta\, d\theta = \int d\theta = \theta + c = \underline{\sin^{-1}\left(\frac{x}{3}\right) + C}$$