warm-up

$$\int \frac{4}{x\sqrt{x^6-1}}\,dx$$

Stuck? think: $\int \frac{1}{u\sqrt{u^2-1}}\,du$

start

$\boxed{u = x^3} \longrightarrow u^2 = x^6$

$du = 3x^2\,dx$

$\boxed{\frac{1}{3x^2}\,du = dx}$

$$\parallel$$

$$4\int \frac{1}{x\sqrt{u^2-1}} \cdot \frac{1}{3x^2}\,du = \frac{4}{3}\int \frac{1}{x^3\sqrt{u^2-1}}\,du = \frac{4}{3}\int \frac{1}{u\sqrt{u^2-1}}\,du$$

$$= \frac{4}{3}\sec^{-1}(u) + c$$

$$= \boxed{\frac{4}{3}\sec^{-1}(x^3) + C}$$

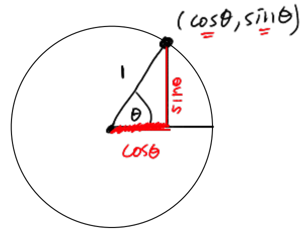
TRIG IDENTITIES REVIEW

pythagorean:

$\sin^2\theta + \cos^2\theta = 1$

$1 + \cot^2\theta = \csc^2\theta$

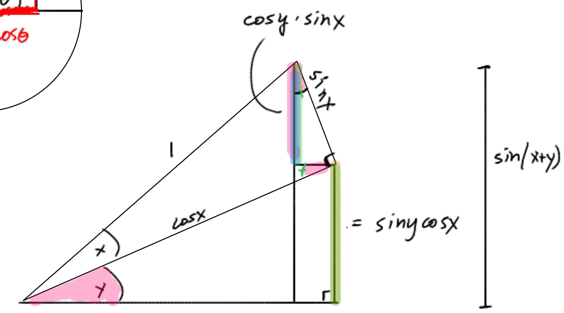
$\tan^2\theta + 1 = \sec^2\theta$



angle sum:

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

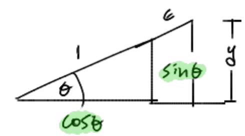


double angle:

$\sin(2a) = 2\sin(a)\cos(b)$

$\cos(2a) = \cos^2(a) - \sin^2(a)$

(from angle sum)



$\sin\theta = \frac{y}{1+\epsilon}$ or $y = \sin\theta \cdot (1+\epsilon)$

$\cos(2a) = 1 - 2\sin^2 a$

$\cos(2a) = 2\cos^2(a) - 1$

half-angle:

$$\sin^2(a) = \frac{1-\cos(2a)}{2}$$

$$\cos^2(a) = \frac{1+\cos(2a)}{2}$$

Trig Integrals

these integrals appear in the study of polar, cylindrical & sperical coord systems.

mercator projections

Ex: $\int \sec\theta \, d\theta = \int \sec\theta \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta} d\theta = \int \frac{\sec^2\theta + \sec\theta\tan\theta}{\sec\theta + \tan\theta} d\theta$

$u = \sec\theta + \tan\theta$

$du = \sec\theta\tan\theta + \sec^2\theta$

$= \boxed{\ln|\sec\theta + \tan\theta| + C}$

Ex: $\int \sin^2\theta \, d\theta = \int \frac{1-\cos(2\theta)}{2} d\theta = \int \frac{1}{2} d\theta - \frac{1}{4}\int \cos(2\theta) 2 d\theta = \boxed{\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + C}$

Ex: $\int \sin^3\theta \, d\theta = \int (1-\cos^2\theta)\sin\theta \, d\theta = \int \sin\theta \, d\theta - \int \cos^2\theta \sin\theta \, d\theta = -\cos\theta + \frac{\cos^3\theta}{3} + C = \cos\theta\left(\frac{\cos^2\theta}{3} - 1\right)$

check: $\frac{d}{d\theta}(\text{ans}) = -\sin\theta\left(\frac{\cos^2\theta}{3} - 1\right) + \cos\theta\left(\frac{2\cos\theta(-\sin\theta)}{3}\right)$

$= -\frac{\cos^2\theta\sin\theta}{3} + \sin\theta - \frac{2\cos^2\theta\sin\theta}{3} = -\cos^2\theta\sin\theta + \sin\theta = \sin\theta(1-\cos^2\theta)$

$= \sin^3\theta.$

Ex: $\int \cos^3\theta \, d\theta = \int (1-\sin^2\theta)\cos\theta = \int \cos\theta \, d\theta - \int \sin^2\theta \cos\theta \, d\theta = \sin\theta - \frac{\sin^3\theta}{3} + C$

$\sin\theta\left(-\frac{\sin^2\theta}{3} + 1\right)$

check: $\frac{d}{d\theta}(\text{ans}) = \cos\theta\left(-\frac{\sin^2}{3} + 1\right) + \sin\theta\left(\frac{-2\sin\theta\cos\theta}{3}\right) = -\sin^2\theta\cos\theta + \cos\theta = \cos\theta(-\sin^2\theta + 1) = \cos^3\theta$

Ex: $\int \sin^3 x \cos^2 x \, dx = \int \sin x \sin^2 x \cos^2 x \, dx = \int \sin x (1-\cos^2 x)\cos^2 x \, dx = \underbrace{\int \sin x \cos^2 x}_{u=\cos x} - \underbrace{\int \cos^4 x \sin x \, dx}_{u=\cos x}$ easy

$= \boxed{\frac{\cos^3 x}{3} - \frac{\cos^5 x}{5} + C}$

check: $\frac{d}{dx}(\text{ans}) = -\cos^2 x \cdot \sin x + \cos^4 x \sin x$

$= \sin x \cos^2 x(\cos^2 x - 1) = \sin^3 x \cos^2 x$

Ex: $\int \sin^2 x \cos^2 x \, dx = \int \left(\frac{1-\cos(2x)}{2}\right)\left(\frac{1+\cos(2x)}{2}\right) dx = \frac{1}{4}\int 1 - \cos^2(2x) \, dx$ $\quad \boxed{-\frac{1}{8}(2x + \sin(4x)) + C}$ //

$= \frac{1}{4}\int \sin^2(2x) dx = \frac{1}{4}\int \frac{1}{2}(1-\cos(4x)) dx = \frac{x}{8}\left[-\frac{1}{4}\int \cos u \, du\right] = \frac{x}{8} - \frac{1}{4}\sin(4x) + C$

$$\int \tan^2 x\,dx = \int \sec^2 x - 1\,dx$$

$$= \boxed{\tan x - x + c}$$

$\sin^2 + \cos^2 = 1$

$\tan^2 + 1 = \sec^2$

$$\int \sec^3 x \tan^5 x\,dx = \int \sec x \tan x \cdot \sec^2 x \tan^4 x\,dx$$

$$= \int \sec x \tan x \cdot \sec^2 (\sec^2 x - 1)^2$$

$$= \int \sec x \tan x \cdot \sec^2 (\sec^4 x - 2\sec^2 x - 1)$$

$$= \int \sec x \tan x \sec^6 dx - 2\int \sec\tan \sec^4 - \int \sec\tan \sec^2$$

$$= \frac{\sec^7 x}{7} - 2\frac{\sec^5 x}{5} - \frac{\sec^3}{3} + c$$