wk 2 - th

Trig Ints: (even powers of sine, cosine, secx, tanx)

① $\int \sin^2(x)\,dx = \int u^2 \frac{1}{\cos x}\,du = \int \frac{u^2}{\sqrt{1-u^2}}\,du = \int \frac{w+1}{\sqrt{w}}\,dw$

$u = \sin(x)$ | $u^2 = \sin^2 x = 1 - \cos^2 x$ | $w = 1-w^2$

$du = \cos(x)\,dx$ | $1-u^2 = \cos^2 x$ | $w+1$ $\rightsquigarrow$ more difficult!

$\frac{1}{\cos x}du = dx$ | $\sqrt{1-u^2} = \cos x$

$$\int \sin^2(x) = \int \frac{1-\cos(2x)}{2}\,dx = \frac{1}{2}\int 1-\cos(2x)\,dx = \frac{1}{2}\left[x - \frac{1}{2}\sin(2x)\right] = \boxed{\frac{1}{2}x - \frac{1}{4}\sin(2x) + C}$$

$$\frac{1}{2}\int \cos(2x)2\,dx = \frac{1}{2}\int \cos(u)\,du = \frac{1}{2}\sin(u)$$

$u = 2x$

$du = 2dx$ $\quad = \boxed{\frac{1}{2}\sin(2x)}$

② $\int \sin^2(x)\cos^2(x)\,dx = \int \sin^2(x)\left(\frac{1+\cos(2x)}{2}\right)dx$

mixed arguments!

$\int \sin^2(x)(1-\sin^2(x))\,dx$

$= \int \sin^2 x - \sin^4(x)\,dx$

$= \frac{1}{2}x - \frac{1}{4}\sin(2x) - \int \underbrace{\sin^4(x)}\,dx$

$-\int (\sin^2(x))^2$

$-\int \left(\frac{1-\cos(2x)}{2}\right)^2 = \frac{1}{4}\int (1-\cos(2x))^2\,dx$

$= \frac{1}{4}\int \left[1 - 2\cos(2x) + \cos^2(2x)\right]dx$

$= \frac{1}{4}\left[x - \sin(2x) + \frac{1}{2}x + \frac{1}{8}\sin(4x)\right]$

$= -\frac{3}{4}x + \frac{1}{4}\sin(2x) - \frac{1}{32}\sin(4x) + C$

$\int \cos^2(2x)\,dx$

$= \int \frac{1+\cos(4x)}{2}\,dx$

$\frac{1}{2}\int 1 + \cos(4x)\,dx$

$\frac{1}{2}(x + \frac{1}{4}\sin(4x))$

$= \boxed{-\frac{1}{4}x - \frac{1}{32}\sin(4x) + C}$

$\cos(2x) = \cos^2(x) - \sin^2 x$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\sin(4x) = \sin(2(2x)) = 2\sin(2x)\cos(2x)$

$= \boxed{4\sin x\cos x \cdot (\cos^2 x - \sin^2 x)}$

sec(x) & tan(x) integrals

① $\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx$ $\frac{u}{du}$ $\int \frac{du}{u}$ $u = \cos x$ $du = -\sin x$ $= -\int \frac{du}{u} = \boxed{-\ln|\cos(x)| + C}$

② $\int \sec(x)\,dx = \int \sec(x)\cdot 1\,dx = \int \sec(x)\,\frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}\,dx = \int \frac{\sec^2 x + \sec x\cdot\tan x}{\sec x + \tan x}\,dx$

$u = \sec x + \tan x$

$du = \sec x \tan x + \sec^2 x$

$= \int \frac{du}{u} = \ln|\sec x + \tan x| + C$

$\frac{\sin^2 + \cos^2 = 1}{\cos^2}$

$\tan^2 + 1 = \sec^2$

pythag. $\tan^2+1$ $u = \tan x$

③ $\int \sec^4(x)\tan^3(x)\,dx = \int \sec^2(x)\,\tan^3(x)\,\underbrace{\sec^2(x)\,dx}_{du}$

(sec(x) ⟷ even (exploit $\sec^2 x$ is a good du)

④ $\int \sec^4(x)\cdot\tan^3(x)\,dx$

tan(x) ⟷ odd