wk 2 — thurs

$$\int \sin^2 x \cos^2 x \, dx \overset{\text{pythag.}}{=} \int (1 - \cos^2 x)\cos^2 x \, dx = \int \cos^2 x - \cos^4 x \, dx$$

both powers are even

$\cos^2 x = \frac{1 + \cos(2x)}{2}$

$$= \int \cos^2 x \, dx - \int \cos^4 x \, dx$$

$(\cos^2 x)^2 = \left(\frac{1+\cos(2x)}{2}\right)^2 = \frac{1}{4}\left[1 + 2\cos(2x) + \cos^2(2x)\right]$

double-angle

$$= \int \frac{1+\cos(2x)}{2} dx - \int \frac{1}{4}\left[1 + 2\cos(2x) + \cos^2(2x)\right] dx$$

$\frac{1}{2}\sin(2x)$

$\frac{1}{2}\int \cos(u)\, du$

$\frac{1}{2}\int \cos(2x) 2\, dx$

$u = 2x$

$du = 2dx$

$$= \frac{1}{2}\int 1 + \cos(2x)\, dx - \frac{1}{4}\left[x + \sin(2x) + \frac{1}{2}x + \frac{1}{4}\sin(2x)\right]$$

$$= \frac{1}{2}\left[x + \frac{1}{2}\sin(2x)\right]$$

$$= \frac{1}{2}x + \frac{1}{4}\sin(2x) - \frac{1}{4}\left[x + \sin(2x) + \frac{1}{2}x + \frac{1}{4}\sin(2x)\right]$$

$\left(\frac{4}{2} - \frac{2}{4} - \frac{1}{8}\right)x$

see: reduction formulas

$$\boxed{\frac{1}{8}x - \frac{1}{16}\sin(2x)}$$

TRIG INTS w/ sec(x) & tan(x)

① $\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx = \int \frac{\sin x}{u}\cdot\left(\frac{-1}{\sin x}\right)du = -\int \frac{du}{u} = -\ln|\cos x| + C$

$u = \sin x$, $du = \cos u$ | $u = \cos x$, $du = -\sin x\,dx$ | $dx = \frac{-1}{\sin x}\,du$

② $\int \sec(x)\cdot 1\,dx = \int \sec x\,\frac{\sec x + \tan x}{\sec x + \tan x}\,dx = \int \frac{\sec^2 x + \sec x\tan x}{\sec x + \tan x}\,dx$

$u = \sec x + \tan x$

$du = \sec^2 x + \sec x\tan x$

$= \int \frac{du}{u} = \boxed{\ln|\sec x + \tan x| + C}$

③ $\int \sec^2(x)\,dx = \tan x + C$

$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$

$\tan^2 + 1 = \sec^2$

④ $\int \sec^3(x)\,dx = \int \sec(x)\cdot\sec^2 x\,dx = \int \sec(x)\cdot(\tan^2 x + 1)\,dx$ dead end

I.B.P.: $u = \sec(x)$, $dv = \sec^2 x$

$du = \sec x\tan x$, $v = \tan x$

$= \sec x\tan x - \int \sec\cdot\tan^2 x\,dx$

$= \sec x\tan x - \int \sec x\cdot(\sec^2 x - 1)\,dx$

$= \sec x\tan x - \int \sec^3 x - \sec x\,dx = \sec x\tan x - \int \sec^3 x\,dx - \int \sec x\,dx$

$\int \sec^3 x\,dx = \frac{1}{2}[\sec x\tan x - \ln|\sec x + \tan x|]$

$-\ln|\sec x + \tan x|$

$A = \text{something} - A - |\ln|\sim|$

$+A$

$-A$

⑤ $\int \sec^5(x)\,dx = ???$ $= \int \sec^5 x - \int \sec^3 x\,dx$