warm-up:

$$\int \sin^2 x \, dx$$

$$=$$

$$\int \frac{1 - \cos 2x}{2} \, dx$$

$u = 2x$
$du = 2dx$

$$\frac{1}{2}\int dx - \frac{1}{2}\frac{1}{2}\int \underbrace{\cos(2x)\, 2dx}_{\cos(u)du}$$

$$\boxed{\frac{1}{2}x - \frac{1}{4}\sin(2x) + C}$$

seating chart

John

Katie

Emma

Grace

Liam

Oliver

Ryan

Andrea

TRIG Ints

$\int u^n \, du$

I. (you) Review: Basic Computations (eg $\int \cos(u)\,du$, $\int \sin(u)\,du$) & u-sub.

II. Products of $\sin^i(x) \cdot \cos^j(x)$. Integrals of these are common (physics engineering)

① $\int \underbrace{\sin(x)}\,\underbrace{\cos(x)\,dx}$ Recognize u — du relationship

$\boxed{u = \sin(x)}$

$\frac{du}{dx} = \cos(x)$, $\boxed{du = \cos(x)\,dx}$

$= \int u^1 \, du$

$= \frac{u^2}{2} + C = \boxed{\frac{(\sin(x))^2}{2} + C}$

---

or $\left.\begin{array}{l} w = \cos(x) \\ dw = -\sin(x)\,dx \end{array}\right\} = \int w(-1)\cdot dw = -\frac{w^2}{2} + C = \boxed{-\frac{(\cos(x))^2}{2} + D}$

or $\frac{\sin(2a)}{2} = \sin(a)\cos(b)$ | $\underline{\underline{\text{problem}}}$ $\int \frac{1}{2}(\sin(2x))\,dx = \frac{1}{2}\int \sin(2x)\,dx$

$= \frac{1}{4}\int \sin(2x)\cdot 2\,dx$

$= \frac{1}{4}\int \sin(u)\,du$

$\left.\begin{array}{l} u = 2x \\ du = 2dx \end{array}\right\} = -\frac{1}{4}\cos(u) + C = \boxed{-\frac{1}{4}\cos(2x) + C}$

②

$$\int \sin^3(x)\,dx = \int \sin(x)\cdot \sin^2(x)\,dx$$

①

$$\int \sin(x)\cdot \frac{1-\cos(2x)}{2}\,dx$$

②

$$\int \sin(x)\cdot\left(1-\cos^2(x)\right)dx$$

$$= \int \sin(x)\,dx - \int \cos^2(x)\cdot \sin(x)\,dx$$

$u = \cos(x)$

$du = -\sin(x)\,dx$

$$= -\cos(x) + \int u^2\,du$$

$$= -\cos(x) + \frac{u^3}{3}$$

$$= -\cos(x) + \frac{\cos^3(x)}{3} + C$$

---

③

$$\int \sin^2 x\cos^2 x\,dx = \int \left[\frac{1-\cos(2x)}{2}\right]\cdot\left[\frac{1+\cos(2x)}{2}\right]dx$$

Half-Angle Formulas

$$= \frac{1}{4}\int (1-\cos(2x))(1+\cos(2x))\,dx = \frac{1}{4}\int 1-\cos^2(2x)\,dx$$

$$= \frac{1}{4}\int dx - \frac{1}{4}\int \cos^2(2x)\,dx = \frac{1}{4}x - \frac{1}{4}\int \frac{1+\cos(4x)}{2}\,dx$$

repeat $\frac{1}{2}$-angle sub

$$= \frac{x}{4} - \frac{1}{8}\int 1\,dx - \left(\frac{1}{4}\right)\frac{1}{8}\int \underbrace{\cos(4x)\,4\,dx}_{\cos(u)\,du}$$

$$= \frac{x}{4} - \frac{x}{8} - \frac{1}{32}\left(\sin(4x)\right) + C = \boxed{\frac{x}{8} - \frac{1}{32}\left(\sin(4x)\right) + C}$$

(4) $\int \cos^4(x)\,dx = \int \left[\frac{1+\cos(2x)}{2}\right]^2 = \frac{1}{4}\int (1+\cos(2x))^2$

$= \frac{1}{4}\int 1 + 2\cos(2x) + \cos^2(2x)\,dx$

stop here for today

$1 \downarrow$ ez

$2\cos(2x) \downarrow$ $u = 2x$, $\int \cos(u)$

$\cos^2(2x) \downarrow$ ½ angle sub, $\int \frac{1+\cos(4x)}{2}$

This one requires a new idea ——— (also useful for $\int \csc(x)\,dx$

$$\int \sec(x)\,dx = \int \sec(x) \cdot \frac{\sec(x) + \tan(x)}{\sec(x) + \tan(x)}\,dx$$

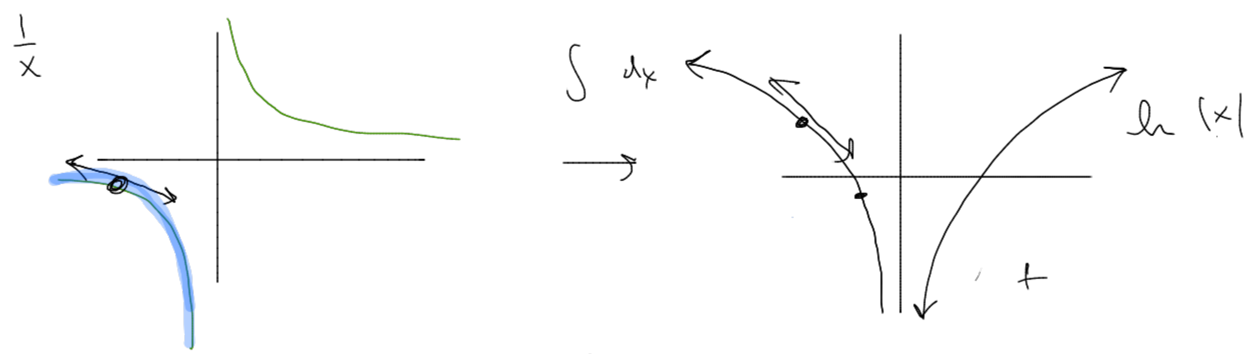
$$= \int \frac{\sec^2(x) + \sec(x)\tan(x)}{\tan(x) + \sec(x)}\,dx$$

$u = \tan(x) + \sec(x)$

$du = \sec^2(x) + \sec(x)\tan(x)\,dx$

$$= \int \frac{du}{u} + C$$

$$= \boxed{\ln\left|\sec(x) + \tan(x)\right| + C}$$



if $\int \frac{1}{x}\,dx = \ln(x)$

$\downarrow$

$\frac{d}{dx}$

$\frac{1}{x}$

Wed. Week 2

warm-up

$$\int \sin^2 x \, dx$$

$$\|$$

$$\int \frac{1-\cos(2x)}{2}\, dx$$

$$\frac{1}{2}\int 1 - \cos(2x)\, dx$$

$$\frac{1}{2}\int 1\, dx - \frac{1}{2}\frac{1}{2}\int \cos(2x)\, 2\, dx$$

$u = 2x$

$du = 2dx$

$$\frac{x}{2} - \frac{1}{4}\int \cos(u)\, du = \boxed{\frac{x}{2} - \frac{1}{4}\sin(2x) + C}$$

seating chart

Brenan

Alex | Tyler

Nicholas

Lauren | Hudson

Josh

Jesse

McKenzie

TRIG INTS

I. Basics ($\int \sin(u)\,du$, $\int \cos(u)\,du$) $\|$ u-sub (You)

II. $\int \sin^i(x)\cos^j(x)\,dx$ (these pop-up in physics / elec engineer)

1 $\int \sin(x)\cos(x)\,dx$

$u = \sin x$

$du = \cos x\,dx$

$= \int u\,du = \frac{u^2}{2} + C = \boxed{\frac{\sin^2(x)}{2} + C}$

or

$u = \cos x$

$-du = \sin x\,dx$

$\int \sin x \cos x\,dx = \int u(-du)$

$= -\int u\,du = -\frac{u^2}{2} + C$

$= \boxed{\frac{-(\cos^2(x))}{2} + C}$

How!

$\sin^2 x = 1 - \cos^2 x$

or

double angle

$\int \sin(x)\cos(x)\,dx = \frac{1}{2}\int \frac{1}{2}\sin(2x)\,2dx = \frac{1}{4}\int \sin(u)\,du$

$u = 2x$

$du = 2dx$

$= -\frac{1}{4}\cos(u) + C$

$= \boxed{-\frac{1}{4}\cos(2x) + C}$

(2) $\int \sin^2(x)\cos(x)\,dx =$

Recognize single power of cos (x) — it can be absorbed into du w/

$u = \sin(x)$
$du = \cos(x)\,dx$

$= \int u^2\,du = \frac{u^3}{3} + C$

$= \boxed{\frac{\sin^3(x)}{3} + C}$

$\int \frac{1-\cos(2x)}{2} \cdot \cos(x)\,dx$

dead end

(3) $\int \sin^2(x)\cos^2(x)\,dx$

$=$

$\int \left[\frac{1-\cos(2x)}{2}\right] \cdot \left[\frac{1+\cos(2x)}{2}\right] dx$

$\frac{1}{4}\int (1-\cos(2x))(1+\cos(2x))\,dx = \frac{1}{4}\int 1 - \cos^2(2x)\,dx$

$= \frac{1}{4}\int 1\,dx - \frac{1}{4}\int \cos^2(2x)\,dx$

$\frac{x}{4} - \frac{1}{4}\int \frac{1+\cos(4x)}{2}\,dx$

$\frac{x}{4} - \frac{1}{8}\int 1 + \cos(4x)\,dx$

$\frac{x}{4} - \frac{x}{8} - \frac{1}{8}\frac{1}{4}\int \cos(4x)\,4\,dx$

$u = 4x$
$du = 4\,dx$

$\frac{x}{4} - \frac{x}{8} - \frac{1}{32}\int \cos(u)\,du$

$\boxed{\frac{x}{8} - \frac{1}{32}\sin(4x) + C}$

or $1 - \cos^2(\heartsuit) = \sin^2(\heartsuit)$

$= \frac{1}{4}\int \sin^2(2x)\,dx$

$= \boxed{\frac{1}{4}\left[\frac{x}{2} - \frac{1}{4}\sin(2x)\right] + C}$

This one involves a new idea

$$\int \sec(x)\,dx = \int \sec(x)\,\frac{\sec(x) + \tan(x)}{\sec(x) + \tan(x)}\,dx$$

$$= \int \frac{\sec^2(x) + \sec(x)\tan(x)}{\sec(x) + \tan(x)}\,dx$$

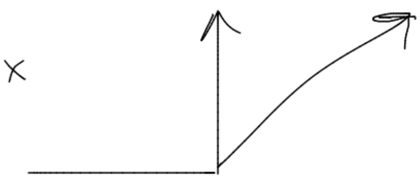
$u = \sec(x) + \tan(x)$

$du = \sec x \tan x + \sec^2 x$

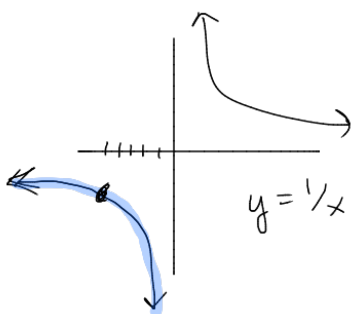
see any derivative relationships

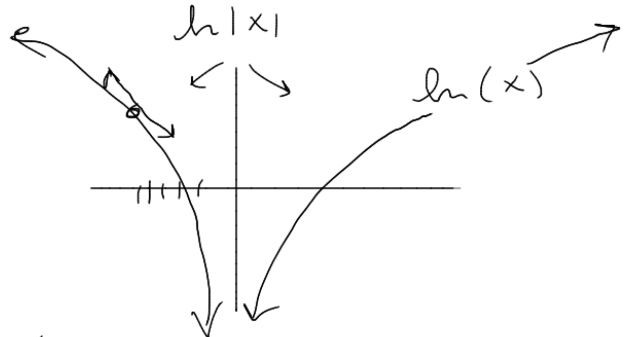
$$= \int \frac{du}{u} = \boxed{\ln\left|\sec(x) + \tan(x)\right| + C}$$

<u>Note</u>: this works for $\int \csc(x)\,dx$



<u>Question</u>: why the abs value?





$$\int \frac{1}{x}\,dx = \ln(x)$$

$$\Big\downarrow d/dx$$

$$\frac{1}{x} = \frac{d}{dx}\left(\ln(x)\right)$$

$$\int \sec^3(x)\tan(x)\,dx$$

$$=$$

$$\int \sec^2(x) \cdot \sec(x)\tan(x)\,dx$$

$u = \sec(x)$

$$=$$

$$\int u^2\,du$$

$$= \frac{u^3}{3} + C$$

$$= \boxed{\frac{\sec^3 x}{3} + C}$$

potential ideas:

1. $u = \tan x$ (fails b/c $du = \sec^2 x$ & we'll have an extra $\sec(x)$)

2. $u = \sec(x)$
   $du = \sec(x)\tan(x)$
   could this work?

3. $\sec^2 x - 1 = \tan^2 x$
   (doubt it)