Wk 2 W

TRIG INTEGRALS

Most can be solved w/ substitution (I.B.P, u= , trig ID)

MAIN TRIG ID's:

Pythagorean

$$\boxed{\sin^2 x + \cos^2 x = 1}$$

÷ by $\sin^2 x$

$$1 + \cot^2 x = \csc^2 x$$

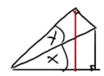
÷ by $\cos^2 x$

$$\tan^2 x + 1 = \sec^2 x$$

Trig Sum Formula

$$\sin(x+y) = \sin(x)\cos(y) + \sin(y)\cos(x)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$



Other Common

(set x=y)

$$\boxed{\begin{aligned} \sin(2x) &= 2\sin x\cos x \\ \cos(2x) &= \cos^2 x - \sin^2 x \end{aligned}}$$

use Pythag

$$\cos(2x) = 1 - 2\sin^2 x$$

isolate $\sin^2 x$

$$\boxed{\sin^2 x = \frac{1-\cos(2x)}{2}}$$

pythag

$$\cos(2x) = \cos^2 x - (1-\cos^2 x)$$

$$= 2\cos^2 x - 1$$

$$\boxed{\cos^2 x = \frac{1+\cos(2x)}{2}}$$

Integrals of form $\int \sin^n(x)\cos^m(x)\,dx$

① $\int \sin(x)\cdot\cos(x)\,dx = \int u\,du = \frac{u^2}{2} + C = \frac{\sin^2(x)}{2} + C$

$u = \sin x$

$du = \cos x\,dx$

$= \frac{1-\cos(2x)}{4} = \frac{1}{4} - \frac{\cos(2x)}{4}$

or

$\int \frac{\sin(2x)}{2}\,dx$ | $u = 2x$, $du = 2\,dx$, $\frac{1}{2}du = dx$

$\frac{1}{2}\int \sin(2x)\,dx = \frac{1}{2}\int \sin(u)\,\frac{1}{2}du = \frac{1}{4}\int \sin(u)\,du = -\frac{1}{4}\cos(u) + C = -\frac{1}{4}\cos(2x) + C$

② $\int \sin^2(x)\cos(x)\,dx = \int u^2\,du = \frac{u^3}{3} + C = \frac{\sin^3 x}{3} + C$

$u = \sin(x)$

$du = \cos x\,dx$

③ $\int \sin^4(x)\cos^3(x)\,dx = \int \sin^4(x)\cos^2(x)\cos(x)\,dx$

think $u = \sin$, $du = \cos$

(*) Pyings $= \int \sin^4(x)\left(1 - \sin^2(x)\right)\cos(x)\,dx$

distribute

$= \int \left[\sin^4(x) - \sin^6(x)\right]\cos(x)\,dx$

dist.

$= \int \sin^4(x)\cos x - \sin^6 x\cos x\,dx$

$= \int \sin^4 x\cos x\,dx - \int \sin^6 x\cos x\,dx$

$u = \sin x$

$du = \cos x\,dx$

$= \int u^4\,du - \int u^6\,du = \frac{u^5}{5} - \frac{u^7}{7} = \boxed{\frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + C}$

Important thing to notice: ODD POWER. peel off a single power and make that part of du

(*) $\int u^4(1-u^2)\,du$ | sub early!

$= \int u^4 - u^6\,du$

④ $\int \sin^2(x)\cos^5(x)\,dx$

odd power of cosine, strip off use the single power as du …. 4th power is a square squared

odd! $= \int \sin^2(x)\cos^4(x)\cos x\,dx$

$\cos^4(x) = (\cos^2 x)^2$

$= \int \sin^2(x)(1-\sin^2 x)^2\cdot\cos x\,dx$

$u = \sin x \quad du = \cos x\,dx$

$= \int u^2(1-u^2)^2\,du = \int u^2(1-2u^2+u^4)\,du = \int u^2 - 2u^4 + u^6\,du$

$= \frac{u^3}{3} - \frac{2u^5}{5} + \frac{u^7}{7} + C = \frac{\sin^3 x}{3} - \frac{2\sin^5 x}{5} + \frac{\sin^7 x}{7} + C$

$$\int \sin^5 x \cos^2 x \, dx$$

" odd!

push $\sin x$ to end $\Rightarrow$ du

$u = \cos x$

$$= \int \sin^4 x \cdot \cos^2 x \cdot \sin x \, dx$$

$\downarrow$

$\cancel{(1-\sin^2 x)}$

$$= \int \sin^4 (1 - \sin^2 x) \sin x \, dx$$

dead end.

$$= \int (\sin^2 x)^2 \cdot \cos^2 x \cdot \sin x \, dx$$

" $(1-\cos^2 x)^2$

$$= \int (1 - 2\cos^2 x + \cos^4 x) \cdot \cos^2 x \cdot \sin x \, dx$$

$$= \int (\cos^2 x - 2\cos^4 x + \cos^6 x) \sin x \, dx$$

$u = \cos x$

$du = -\sin x \, dx$

$$= -\int u^2 - 2u^4 + u^6 \, du = -\frac{u^3}{3} + \frac{2u^5}{5} - \frac{u^7}{7} + C$$

$$= -\frac{\cos^3 x}{3} + \frac{2\cos^5 x}{5} - \frac{\cos^7 x}{7} + C$$