Wk 2 W

TRIG INTEGRALS (7-2)

Most trig ints can be solved w/ substitution ($u =$ ______ , trig identities)

Main Trig Identies

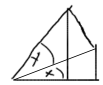
Pythagorean

- $\cos^2 x + \sin^2 x = 1$

$1 + \tan^2 x = \sec^2 x$ ← $\div$ by $\cos^2 x$

$\cot^2 x + 1 = \csc^2 x$ ← $\sin^2 x$

Trig Sum



- $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$
- $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

set $x=y$

Other Common

$\boxed{\sin(2x) = 2\sin(x)\cos(x)}$

$\boxed{\cos(2x) = \cos^2 x - \underbrace{\sin^2 x}_{=(1-\cos^2 x)}}$

pythag.

$= 2\cos^2 x - 1$

$\Rightarrow \boxed{\frac{1+\cos(2x)}{2} = \cos^2 x}$

pythag

$\cos(2x) = 1 - 2\sin^2 x$

$\boxed{\sin^2 x = \frac{1-\cos(2x)}{2}}$

Integrals of form $\int \sin^n(x)\cos^m(x)\,dx$

$\int \sin(x)\cos(x)\,dx = \int \frac{1}{2}\sin(2x)\,dx$ $\quad u = 2x,\ du = 2\,dx,\ \frac{1}{2}du = dx$

$= \frac{1}{2}\int \sin(u)\frac{1}{2}du$

$= -\frac{1}{4}\cos(u) + C = -\frac{1}{4}\cos(2x) + C$

$\boxed{\cos^2 x = \frac{1+\cos(2x)}{2} \qquad \sin^2 x = \frac{1-\cos(2x)}{2}}$

or

$u = \cos(x)$
$du = -\sin(x)\,dx$
$\frac{1}{-\sin x}du\ dx$

$= \int \sin(x)\cdot u\left(\frac{-1}{\sin x}du\right) = -\int u\,du = -\frac{u^2}{2} + C = -\frac{(\cos(x))^2}{2} + C$

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mixed arguments!

$\int \sin^2 x \cos x\,dx \overset{\text{if}}{=} \int \frac{1-\cos 2x}{2}\cdot\cos x\,dx$

$u = \sin x$
$du = \cos x\,dx$

$= \int u^2\,du = \frac{u^3}{3} + C = \frac{\sin^3 x}{3} + C$

$\int \sin^2 x\cdot\cos^3 x\,dx$ $\quad \boxed{\text{odd power!}}$

$= \int \sin^2 x\cdot\underbrace{\cos^2 x}_{=(1-\sin^2 x)}\cdot\cos x\,dx = \int(\sin^2 - \sin^4 x)\cdot\underbrace{\cos x\,dx}_{du} = \int \sin^2 x\cos x - \sin^4 x\cos x\,dx$

idea: $u = \sin x$, $du = \cos x\,dx$ $\quad u = \sin x$, $du = \cos x$

$= \int u^2 - u^4\,du$

$= \int u^2\,du - \int u^4\,du$

$= \frac{u^3}{3} - \frac{u^5}{5} = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$

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odd power!

$\int \sin^5(x)\cos^4(x)\,dx$

$= \int \sin^4(x)\,\underline{\sin(x)}\cos^4(x)\,dx = \int \sin^4 x\cos^4 x\sin(x)\,dx$

peel off a single $\sin(x)$, move to back ... this becomes (part of) du

set $u = \cos(x)$

transform $\sin^4(x)$ even $\sim \cos^2 x$

$= \int(\sin^2 x)^2\cos^4 x\sin(x)$

$= \int(1-\cos^2 x)^2\cos^4 x\sin x\,dx$

$= \int(1 - 2\cos^2 x + \cos^4 x)\cos^4 x\sin x\,dx$

$= \int(\cos^4 x - 2\cos^6 x + \cos^8 x)\sin x\,dx \leftrightarrow$ $u = \cos x$, $du = -\sin x$

$= -\int u^4 - 2u^6 + u^8\,du = -\left(\frac{u^5}{5} - \frac{2u^7}{7} + \frac{u^9}{9}\right) + C = -\left(\frac{\cos^5 x}{5} - \frac{2\cos^7 x}{7} + \frac{\cos^9 x}{9}\right) + C$