Wk 3 Mon

LIATE

Hw Q

Technique: separate power to make dv easy to integrate.

$$\int 7x^7 \cos(x^4)\,dx$$

product $\Rightarrow$ guess I.B.P.

$u = x^4$
$du = 4x^3 dx$

$u = 7x^4$ $\quad dv = \cos(x^4)x^3 dx$

$du = 28x^3$ $\quad = \frac{1}{4}\cos(x^4)4x^3 dx$

$$v = \int dv = \frac{1}{4}\int \cos(u)\,du = \frac{1}{4}\sin(x^4)$$

$$= uv - \int v\,du = \frac{7x^4}{4} \cdot \sin(x^4) - \frac{7}{4}\int \underbrace{\sin(x^4)}_{u} \underbrace{4x^3 dx}_{du}$$

$$= \frac{7}{4}x^4 \sin(x^4) + \frac{7}{4}\cos(x^4) + C$$

TRIG SUBSTITUTION

useful for:

- $\int \sqrt{1-x^2}\,dx$, $\int \sqrt{a^2-x^2}\,dx$, $\int \frac{1}{\sqrt{a^2-x^2}}\,dx$, $\int \frac{1}{(a^2-x^2)^{3/2}}\,dx$,...

  constant minus square under rational exponent

- $\int \frac{1}{1+x^2}\,dx$, $\int \frac{1}{a^2+x^2}\,dx$

  fractions, constant plus square — no radical!

- $\int \frac{1}{\sqrt{x^2-a^2}}\,dx$, $\int \frac{1}{x^2\sqrt{x^2-a^2}}\,dx$,

  square minus constant under rational exponent

Key:

① use domain to inform your substitution

Ex $\sqrt{4-x^2}$ Domain of $[-2,2]$

$\Rightarrow x = 2\sin\theta$, with $\theta \in [\frac{-\pi}{2}, \frac{\pi}{2}]$

Ex $1+x^2$ Domain $\mathbb{R}$ $\Rightarrow x = \sqrt{1}\cdot\tan\theta$
$= \tan\theta$

Ex $\sqrt{x^2-a^2}$ Domain $= (-\infty,-a] \cup [a,\infty)$
$x = a\sec(\theta)$ $\theta \in$ QI, III

Ex

$$\int \frac{1}{\sqrt{9-x^2}}\,dx$$

think $x \in [-3,3]$

① $\Rightarrow$ $\boxed{x = 3\sin\theta \quad \theta \in [-\pi/2, \pi/2]}$

② $dx = 3\cos\theta\, d\theta$

③ sub: $\displaystyle\int \frac{1}{\sqrt{9-(3\sin\theta)^2}}\, dx = \int \frac{1}{\sqrt{\underbrace{9-9\sin^2\theta}_{\sqrt{9(1-\sin^2\theta)} \,=\, \sqrt{9\cos^2\theta}}}} \cdot 3\cos\theta\, d\theta = \int \frac{1}{3\underbrace{|\cos\theta|}_{=\cos\theta}} 3\cos\theta\, d\theta$

$$= \int d\theta = \theta + C$$

④ get x back:

solve ① for θ: $\frac{x}{3} = \sin\theta \Rightarrow \theta = \sin^{-1}\left(\frac{x}{3}\right)$ $= \boxed{\sin^{-1}\left(\frac{x}{3}\right) + C}$

Ex.

$$\int \sqrt{4-x^2}\, dx$$

think: domain $x \in [-2,2]$ $\quad (*)\ \theta \in [-\pi/2, \pi/2]$

$\Rightarrow$ $x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

$$\int \sqrt{\underbrace{4-4\sin^2\theta}} \cdot 2\cos\theta\, d\theta = \int 4\cos^2\theta\, d\theta = 4\int \cos^2\theta\, d\theta$$

$\sqrt{4(1-\sin^2\theta)}$

$\sqrt{4\cos^2\theta}$

$2|\cos\theta| \overset{(*)}{=} 2\cos\theta$

$$= 4\int \frac{1+\cos(2\theta)}{2}\, d\theta = 2\int 1+\cos(2\theta)\, d\theta$$

$\int \cos(2\theta)\, d\theta$

$u = 2\theta$

$du = 2d\theta$

$\frac{1}{2}du = d\theta$

$= \int \cos(u)\frac{1}{2}du$

$= \frac{1}{2}\int \cos u$

$= \frac{1}{2}\sin(2\theta)$

$$= 2\left[\theta + \frac{1}{2}\sin(2\theta)\right] + C = 2\theta + \sin(2\theta) + C$$

adj/hyp

$= 2\sin^{-1}\left(\frac{x}{2}\right) + 2\underbrace{\sin\theta}_{\frac{x}{2}}\underbrace{\cos\theta}_{\frac{\sqrt{4-x^2}}{2}}$

$= \boxed{2\sin^{-1}\left(\frac{x}{2}\right) + \frac{1}{2}\cdot x\sqrt{4-x^2}}$

$\sin(A+B) = \sin A\cos B + \sin B\cos A$

$\sin(2\theta) = \sin\theta\cos\theta + \sin\theta\cos\theta$

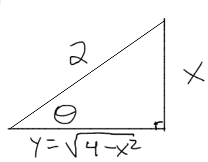
$= 2\sin\theta\cos\theta$

#

get x back

$\frac{x}{2} = \sin\theta$ = opp/hyp

$\sin^{-1}\left(\frac{x}{2}\right) = \theta$



$2^2 = x^2 + y^2$

$4 - x^2 = y^2$

$\sqrt{4-x^2} = y$

create a right triangle that "mirrors" your substitution equation

$x = 2\sin\theta$

use pythagorean to give other side