Quiz, Sections 7.1, 7.2, & 7.3

1.

$$\int x^2 \sin \pi x \, dx = -\frac{1}{\pi} x^2 \cos \pi x + \frac{2}{\pi^2} x \sin \pi x + \frac{2}{\pi^3} \cos \pi x + C$$

sign	u		dv
$+$	x^2		$\sin \pi x$
		$\searrow$	
$-$	$2x$		$-\frac{1}{\pi} \cos \pi x$
		$\searrow$	
$+$	2		$-\frac{1}{\pi^2} \sin \pi x$
		$\searrow$	
	0		$\frac{1}{\pi^3} \cos \pi x$

2. Use $u = \sin \pi x \longrightarrow du = \pi \cos \pi x \, dx$.

$$\int \sin^2(\pi x) cos^5(\pi x) \, dx = \int \frac{1}{\pi} \sin^2 \pi x (1 - \sin^2 \pi x)(1 - \sin^2 \pi x)(\pi \cos \pi x) \, dx$$

$$= \int \frac{1}{\pi} u^2 (1 - u^2)(1 - u^2) \, du = \int \frac{1}{\pi} u^2 - \frac{2}{\pi} u^4 + \frac{1}{\pi} u^6 \, du$$

$$= \frac{1}{3\pi} u^3 - \frac{2}{5\pi} u^5 + \frac{1}{7\pi} u^7 + C$$

$$= \frac{1}{3\pi} \sin^3 \pi x - \frac{2}{5\pi} \sin^5 \pi x + \frac{1}{7\pi} \sin^7 \pi x + C$$

3. Use $x = 3\sin\theta \longrightarrow dx = 3\cos\theta\, d\theta$ and $9 - x^2 = 9\cos^2\theta$.

$$\int \frac{\sqrt{9-x^2}}{x^2}\, dx = \int \frac{\sqrt{9\cos^2\theta}}{(3\sin\theta)^2}\,(3\cos\theta)\; d\theta = \int \frac{\cos^2\theta}{\sin^2\theta}\, d\theta$$

$$= \int \cot^2\theta\, d\theta = \int \csc^2\theta - 1\, d\theta = -\cot\theta - \theta + C$$

$$= -\frac{\sqrt{9-x^2}}{x} - \sin^{-1}\left(\frac{x}{3}\right) + C$$