

**some standard trig substitution problems**

1. Use  $x^2 = 4 \sin^2 \theta \implies x = 2 \sin \theta$ , then  $dx = 2 \cos \theta d\theta$

$$\begin{aligned}\int \sqrt{4-x^2} dx &= \int \sqrt{4-4\sin^2 \theta} (2 \cos \theta) d\theta \\ &= \int \sqrt{4\cos^2 \theta} (2 \cos \theta) d\theta = \int 4 \cos^2 \theta d\theta \\ &= \int 4 \cdot \frac{1}{2} (1 + \cos 2\theta) d\theta = \int 2 + 2 \cos 2\theta d\theta \\ &= 2\theta + \sin 2\theta + C = 2\theta + 2 \sin \theta \cos \theta + C \\ &= 2 \sin^{-1} \left( \frac{x}{2} \right) + \frac{x\sqrt{4-x^2}}{2} + C\end{aligned}$$

2. Use  $4x^2 = \tan^2 \theta \implies 2x = \tan \theta$ , then  $dx = \frac{1}{2} \sec^2 \theta d\theta$ .

$$\begin{aligned}\int \frac{1}{\sqrt{4x^2+1}} dx &= \int \frac{1}{\sqrt{\tan^2 \theta + 1}} \cdot \frac{1}{2} \sec^2 \theta d\theta \\ &= \frac{1}{2} \int \frac{\sec^2 \theta}{\sqrt{\sec^2 \theta}} d\theta = \frac{1}{2} \int \sec \theta d\theta \\ &= \frac{1}{2} \ln |\sec \theta + \tan \theta| + C \\ &= \frac{1}{2} \ln |2x + \sqrt{4x^2+1}| + C\end{aligned}$$

3. Use  $x^2 = 9 \sec^2 \theta \implies x = 3 \sec \theta$ , then  $dx = 3 \sec \theta \tan \theta d\theta$ .

$$\begin{aligned}\int \frac{1}{\sqrt{x^2 - 9}} dx &= \int \frac{1}{\sqrt{9 \sec^2 \theta - 9}} (3 \sec \theta \tan \theta) d\theta \\ &= \int \frac{1}{\sqrt{9 \tan^2 \theta}} (3 \sec \theta \tan \theta) d\theta = \int \sec \theta d\theta \\ &= \ln |\sec \theta + \tan \theta| + C \\ &= \ln \left| \frac{x}{3} + \frac{\sqrt{9 - x^2}}{3} \right| + C\end{aligned}$$

4. Use  $x^2 = 9 \sin^2 \theta \implies x = 3 \sin \theta$ , then  $dx = 3 \cos \theta$ .

$$\begin{aligned}\int \frac{1}{\sqrt{9 - x^2}} dx &= \int \frac{1}{\sqrt{9 - 9 \sin^2 \theta}} (3 \cos \theta) d\theta \\ &= \int \frac{1}{\sqrt{9 \cos^2 \theta}} (3 \cos \theta) d\theta = \int 1 d\theta \\ &= \theta + C = \sin^{-1} \left( \frac{x}{3} \right) + C\end{aligned}$$