Fri - wk 3

Exam 1 : Wed

see Notes Page

Partial Fraction technique:

- Improper Fractions : (degree of numerator $\geq$ degree denom.)
- key: long division

$$\int \frac{x^2 - 7x + 1}{x^2 + x}\, dx = \int 1 + \frac{-8x+1}{x^2+x}\, dx = x + \int \frac{-8x+1}{\underbrace{x^2+x}_{x(x+1)}}\, dx = x \int \frac{A}{x} + \frac{B}{x+1}\, dx$$

$$\begin{array}{r} 1 \\ x^2 + x \overline{)\, x^2 - 7x + 1} \\ -(x^2 + x) \\ \hline -8x + 1 \end{array}$$

stop here b/c remainder degree < divisor degree

clear: $-8x+1 = A(x+1) + B(x)$

sub: $x = -1 \Rightarrow -8(-1)+1 = A(0) + B(-1)$

$9 = B(-1) \Rightarrow B = -9$

$x = 0 \Rightarrow 1 = A$

$$= x + \int \frac{1}{x} - \frac{9}{x+1}\, dx = \boxed{x + \ln|x| - 9\ln|x+1| + C}$$

$u = x+1$

$du = dx$

Irreducible Case

$$\int \frac{29x^2}{(x+1)(x^2+1)}\,dx = \int \frac{A}{x+1} + \frac{Bx+C}{x^2+1}\,dx = \int \frac{14.5}{x+1}\,dx + \overbrace{\int \frac{14.5x-14.5}{x^2+1}\,dx}^{14.5\int \frac{x-1}{x^2+1}dx}$$

irreducible

clear: $29x^2 = A(x^2+1) + (Bx+C)(x+1) = Ax^2 + A + Bx^2 + Bx + Cx + C$

$= (A+B)x^2 + (B+C)x + (A+C)$

x^2: $29 = A+B \Rightarrow 2A$

x: $B+C=0$ $\quad B=-C$

const $A+C=0$, $\quad A=-C$ $\Big\}$ $A=B$

$A = \frac{29}{2} = 14.5$

$B = 14.5$

$C = -14.5$

$-\ln\left((x^2+1)^{-1/2}\right) = \frac{1}{2}\ln(x^2+1)$

$$= 14.5\ln|x+1| + 14.5\left(\underbrace{-\ln\left|\frac{1}{\sqrt{x^2+1}}\right|} - \tan^{-1}(x)\right) + c$$

$$\boxed{\int \frac{x-1}{x^2+1}\,dx = \int \frac{\tan\theta - 1}{\sec^2\theta}\sec^2\theta\,d\theta = \int \tan\theta - 1\,d\theta = -\ln|\cos\theta| - \theta}$$

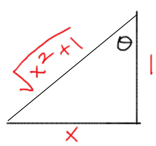
$= -\ln\left|\frac{1}{\sqrt{x^2+1}}\right| - \tan^{-1}\left(\frac{x}{1}\right)$

$x = \tan\theta = \frac{x}{1}$

$x^2 = \tan^2\theta + 1 = \sec^2\theta$

$x - 1 = \tan\theta - 1$

$dx = \sec^2\theta\,d\theta$



$u = x^2+1$

$du = 2x\,dx$

$$\int \frac{x}{x^2+1} - \frac{1}{x^2+1}\,dx = \frac{1}{2}\ln|x^2+1| - \tan^{-1}(x)$$

Irreducible

$$\int \frac{1}{x^2+x+1}\,dx = \int \frac{Ax+B}{x^2+x+1}\,dx$$

clear $1 = Ax + B$ stuck

sol'n

Complete the $\square$

$$\int \frac{1}{x^2+x+\frac{1}{4}-\frac{1}{4}+1}\,dx = \int \frac{1}{(x+\frac{1}{2})^2+\frac{3}{4}}\,dx$$

Proceed as usual —

$b=1$, add / subtract $\left(\frac{-b}{a}\right)^2 = \frac{b^2}{4} = \frac{1}{4}$

$(x+\frac{1}{2})^2 = x^2 + x + \frac{1}{4}$

$$\int \frac{x^5}{\sqrt{1-4x^2}}\,dx =$$

$a^2 - x^2$

$x^2 - a^2$

$1 + x^2$

want

$\sqrt{1-4x^2} = \sqrt{4\left(\frac{1}{4} - x^2\right)} = 2\sqrt{\left(\frac{1}{2}\right)^2 - x^2} = \sqrt{a^2 - 0x^2}$

$a = \frac{1}{2}$

$$\int x^4 e^{3x}\,dx = \frac{x^4}{3}e^{3x} - \int 4x^3 \cdot \frac{1}{3}e^{3x}\,dx$$

$+$	$u = x^4$	$dv = e^{3x}$
$-$	$du = 4x^3$	$v = \frac{1}{3}e^{3x}$
$+$	$du = 12x^2$	$v = \frac{1}{9}e^{3x}$
$-$	$du = 24x$	$v = \frac{1}{27}e^{3x}$
$+$	$du = 24$	$v \quad \frac{1}{81}e^{3x}$
	$du = 0$	$v = \frac{1}{243}e^{3x}$