MA163 wk 3 Fri

practial fraction denoms

$$\int \frac{x^2 - 11x + 1}{x^2 + x} dx = \int 1 + \frac{-12x+1}{x^2+x} dx = x + \int \frac{-12x+1}{\underbrace{x^2+x}_{x(x+1)}} dx = x + \int \frac{A}{x} + \frac{B}{x+1} dx$$

partial fraction technique needs the rational functions in “proper form” … degree of numerator < degree of denom.

$$\begin{array}{r|l} & \quad \textcircled{1} \\ \hline x^2 + x & x^2 - 11x + 1 \\ & -(x^2 + x) \\ \hline & \quad -12x + 1 \end{array}$$

$\leftarrow$ Remainder degree < divisor degree

$\Rightarrow$ Stop

$(A+B)x + A$

① clear: $-12x + \underline{\underline{1}} = A(x+1) + Bx = Ax + Bx + A$

② $x = -1 \Rightarrow 12 + 1 = A(0) - B \Rightarrow B = -13$

equate coef: $A = 1$

$$\Rightarrow = x + \int \frac{1}{x} - \frac{13}{x+1} dx = \boxed{x + \ln|x| - 13\ln|x+1| + C}$$

$\int \frac{1}{x+1} dx$ u-sub $\uparrow$

Ex.

$$\int \frac{(5x-3)\,dx}{x^2-13x+42} = \int \frac{5x-3}{(x-6)(x-7)}\,dx = \int \frac{A}{x-6} + \frac{B}{x-7}\,dx = \int \frac{-27}{x-6} + \frac{32}{x-7}\,dx$$

$$= \boxed{-27\ln|x-6| + 32\ln|x-7| + c}$$

see: Rational ⇒ Partial Fraction

<u>clear</u>:

$$5x-3 = \underline{A}(x-7) + \underline{B}(x-6) = (A+B)x + (-7A-6B)$$

$$\begin{aligned} x=7:&\quad 35-3 = \boxed{B = 32} \\ x=6:&\quad 30-3 = \boxed{A = -27} \end{aligned}$$

$$\boxed{\begin{aligned} A+B&=5 \\ -7A-6B&=-3 \end{aligned}} \rightarrow$$

Exam 1 Guide

1.

$$\int x^4 e^{3x}\, dx =$$

2.

$$\int \frac{4x-1}{x^2-5x-14}\, dx =$$

3. u = sub

$$\int x^4 \sec^2(x^5)\ dx =$$

$$\int x^3 e^{2x}\,dx = \frac{x^3}{2}e^{2x} - \int 3x^2 \tfrac{1}{2}e^{2x}\,dx = \overset{\overset{+}{\downarrow}}{\frac{x^3}{2}e^{2x}} \overset{\overset{-}{\downarrow}}{-} \left[\overbrace{\frac{3x^2}{4}e^{2x} - \underbrace{\int 6x\cdot\tfrac{1}{4}e^{2x}\,dx}}^{\overset{+}{uv} - \overbrace{\int v\,du}^{uv - \int v\,du}}\right]$$

I.B.P.

sign

$+$ $u = x^3$ $\qquad dv = e^{2x}$

$-$ $u_2 = du = 3x^2$ $\qquad v = \frac{1}{2}e^{2x}$

$+$ $du = 6x$ $\qquad v = \frac{1}{4}e^{2x}$

$-$ $du = 6$ $\qquad v = \frac{1}{8}e^{2x}$

$du = 0$ $\qquad v = \frac{1}{16}e^{2x}$

$$\boxed{\frac{x^3}{2}e^{2x} - \frac{3x^2}{4}e^{2x} + \frac{6x}{8}e^{2x} - \frac{6}{16}e^{2x} + C}$$