Monday - Week 3

$u = \sec\theta + \tan\theta$
$du = \sec^2\theta + \sec\theta\tan\theta \, d\theta$

warm-up

(a) $\int \sec(\theta)\,d\theta = \int \sec\theta \cdot \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}\,d\theta = \int \frac{\sec^2\theta + \sec\theta\tan\theta}{\sec\theta + \tan\theta}\,d\theta$

$= \int \frac{du}{u} = \ln|u| = \ln|\sec\theta + \tan\theta| + C$

(b) $\int \sec^3(\theta)\,d\theta \quad \int \sec^2\theta \sec\theta\,d\theta = \int (\tan^2\theta + 1)\sec\theta\,d\theta$

$= \int \tan^2\theta \sec\theta\,d\theta + \underbrace{\int \sec\theta\,d\theta}_{\text{part (a)}}$

$\left\{ \tan^2\theta = \sec^2\theta - 1 \right.$ leads to a dead end $\}$

$= \underbrace{\int \tan\theta \tan\theta \sec\theta\,d\theta}$ + (part (a))

$u = \tan\theta$ | $dv = \sec\theta\tan\theta\,d\theta$

$du = \sec^2\theta$ | $v = \sec\theta$

$= \sec\theta\tan\theta - \int \sec^3\theta\,d\theta$ + (part (a))

$$\boxed{\int \sec^3\theta = \frac{\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|}{2}}$$

TRIG · SUBSTITUTION

Ex
$$\int \sqrt{9-x^2}\,dx$$
$$\updownarrow$$
$$\sqrt{3^2-x^2}$$
$$\updownarrow$$
$$\sqrt{a^2-x^2}$$
$(a=3)$

① Domain $\sqrt{9-x^2}$ is $[-3,3]$

Replace x w/ a trig function whose Range is $[-3,3]$

$$x = a\cdot\sin\theta$$
$$\boxed{x = 3\sin\theta}$$

② $$\boxed{\begin{aligned} dx &= 3\cos\theta\, d\theta \\ x^2 &= 9\sin^2\theta \end{aligned}}$$ square

③ Substitute:

$$\int \sqrt{9-9\sin^2\theta}\cdot 3\cos\theta\, d\theta = 3\int \sqrt{9(1-\sin^2\theta)}\,\cos\theta\, d\theta$$
$$= 9\int \sqrt{1-\sin^2\theta}\cdot\cos\theta\, d\theta$$
$$= 9\int \sqrt{\cos^2\theta}\,\cos\theta\, d\theta$$
$$= 9\int \cos^2\theta\, d\theta$$

④ $$= 9\int \frac{1+\cos(2\theta)}{2}\,d\theta = \frac{9}{2}\left[\int 1\,d\theta + \int \cos(2\theta)\,d\theta\right]$$

$$= \frac{9}{2}\theta + \frac{1}{2}\cdot\frac{9}{2}\int \cos(2\theta)2\,d\theta \qquad u = 2\theta$$

$\sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a)$

$\theta = a = b$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$$= \frac{9}{2}\theta + \frac{9}{4}\int \cos(u)\,du$$
$$= \frac{9}{2}\theta + \frac{9}{4}\sin(2\theta) + C$$
$$= \frac{9}{2}\theta + \frac{9}{2}\sin\theta\cos\theta + C$$

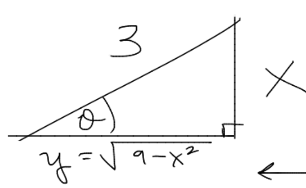
SOHCAHTOA

⑤ $$\boxed{x = 3\sin\theta}$$

if:

$$\frac{x}{3} = \sin\theta$$

$$\frac{\sqrt{9-x^2}}{3} = \cos\theta$$



pythagoras

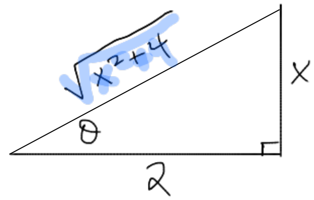
$y^2 + x^2 = 3^2$

$y = \sqrt{9-x^2}$

$$\sin^{-1}\left(\frac{x}{3}\right) = \theta$$

⑥ $$= \frac{9}{2}\cdot\sin^{-1}\left(\frac{x}{3}\right) + \frac{9}{2}\cdot\frac{x}{3}\cdot\frac{\sqrt{9-x^2}}{3} = \boxed{\frac{9\sin^{-1}\left(\frac{x}{3}\right)}{2} + \frac{x\sqrt{9-x^2}}{2} + C}$$

② $\int x^3\sqrt{x^2+4}\,dx$



(0) u-sub fails

(1) Domain: $\sqrt{x^2+4} = \mathbb{R}$

$x = a\tan x$ $\quad\sqrt{x^2+a^2}$

$a = 2$

$\boxed{x = 2\tan\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\cos\theta = \frac{adj}{hyp}$

$\sec\theta = \frac{hyp}{adj} = \frac{\sqrt{x^2+4}}{2}$

(2) $dx = 2\sec^2\theta\,d\theta$

$x^2 = 4\tan^2\theta$

$x^3 = 8\tan^3\theta$ $\qquad 2\sqrt{\tan^2\theta+1} = 2\sqrt{\sec^2\theta} = 2\sec\theta$

〃

(3) ANS $= \int 8\tan^3\theta\sqrt{4\tan^2\theta+4}\cdot 2\sec^2\theta\,d\theta$

$= 32\int\tan^3\theta\sec^3\theta\,d\theta$

$= 32\int\tan\theta\tan^2\theta\sec^2\theta\cdot\sec\theta\,d\theta$

$= 32\int(\sec^2\theta-1)\sec^2\theta\cdot\sec\theta\tan\theta\,d\theta$

$= 32\int(\sec^4\theta-\sec^2\theta)\sec\theta\tan\theta\,d\theta$

$u = \sec\theta$

$du = \sec\theta\tan\theta\,d\theta$

$= 32\int u^4 - u^2\,du$

$= \frac{32u^5}{5} - \frac{32u^3}{3} + c$

$= \frac{32\sec^5\theta}{5} - \frac{32\sec^3\theta}{3} + c$

$= \frac{32\left(\frac{\sqrt{x^2+4}}{2}\right)^5}{5} - \frac{32\left(\frac{\sqrt{x^2+4}}{2}\right)^3}{3} + c$