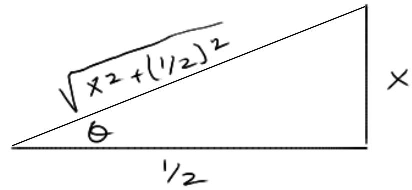
Warm-up 10am section pages 1-4

$$\int \frac{1}{\sqrt{4x^2+1}}\,dx = \int \frac{1}{\sqrt{4x^2+4/4}}\,dx = \int \frac{1}{2\sqrt{x^2+1/4}}\,dx = \int \frac{1/2}{\sqrt{x^2+(1/2)^2}}\,dx$$

$a = 1/2$, $x = \frac{1}{2}\tan\theta$, $dx = \frac{1}{2}\sec\theta$

$\frac{x}{1/2} = \tan\theta$ $\qquad \theta = \tan^{-1}(2x)$



$$= \int \cos\theta \cdot \tfrac{1}{2}\sec\theta\, d\theta = \tfrac{1}{2}\int d\theta = \theta/2 + C = \frac{\tan^{-1}(2x)}{2} + C$$

check: $\frac{d}{dx}(\text{ans}) = \frac{1}{2}\,\frac{1}{\sqrt{1+4x^2}}\cdot 2\,dx = \frac{1}{\sqrt{1+4x^2}}\,dx$:)

- complete square ?
- ellipse derivation

Week 3 — Thursday

LIPET

WW #2

$$\int 6\sin(\sqrt{x})\,dx = 6\int \sin(u)\cdot 2u\,du = 12\int u\cdot\sin(u)\,du$$

1. $\boxed{u=\sqrt{x}}$

$du = \frac{1}{2}x^{-1/2}\,dx$

$2x^{1/2}\,du = dx$

$\boxed{2u\,du = dx}$

$u = u$	$dv = \sin(u)$
$du = du$	$v = -\cos(u)$

$$= 12\left[-u\cdot\cos(u) + \int \cos(u)\,du\right]$$

$$= 12\left[-u\cdot\cos(u) + \sin(u)\right]$$

$$= 12\left[-\sqrt{x}\cdot\cos(\sqrt{x}) + \sin(\sqrt{x})\right] + C$$

#5

$$7\int x^3 \cdot e^{-x^2}\,dx \xrightarrow{\text{I.B.P.}} 7\int x^2 \cdot x \cdot e^{-x^2}\,dx = 7\left[\frac{-x^2}{2}\cdot e^{-x^2} - \int\left(-\frac{1}{2}e^{-x^2}\cdot 2x\,dx\right)\right]$$

blue terms = du

$= -\frac{1}{2}\int e^u\,du$

$u = -x^2$

$du = -2x\,dx$

$$= 7\left[\frac{-x^2 e^{-x^2}}{2} - \frac{1}{2}e^{-x^2}\right] + C$$

$$7\left[\frac{-e^{-x^2}\left[x^2+1\right]}{2}\right] + C$$

$u = x^3$

$du = 3x^2\,dx$

$dv = e^{-x^2}$

$v =$ can't integrate

Restart

$\boxed{u = x^2}$

$\boxed{du = 2x\,dx}$

$\boxed{dv = e^{-x^2}\cdot x\,dx}$

$v = \int dv = -\frac{1}{2}\int e^{-x^2}\underbrace{(-2)\cancel{x}\,dx}_{dw}$

$w = -x^2$

$dw = -2x\,dx$

$\boxed{\frac{-1}{2}\,dw = x\,dx}$

$= \int e^w\left(-\frac{1}{2}\,dw\right)$

$= -\frac{1}{2}\int e^w\,dw$

$= -\frac{1}{2}e^w = \boxed{-\frac{1}{2}e^{-x^2} = v}$

so dx

$dx = \frac{-1}{2x}\,dw$

then

$\int e^{-x^2}\cancel{x}\cdot\left(\frac{-1}{2\cancel{x}}\right)dw$

$= -\frac{1}{2}\int e^w\,dw$

Last Time:

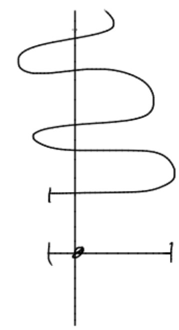
$$\int_{-3}^{3} \sqrt{9-x^2}\,dx = \tfrac{1}{2} \text{ area of a circle w/ radius } = 3$$

$$\text{Area}(\bigcirc) = \pi(3)^2 = 9\pi$$

$$= \frac{9\pi}{2}$$

Verify this by using trig substitution

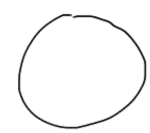
$$\left[-\tfrac{\pi}{2}, \tfrac{\pi}{2}\right]$$



Domain of $\sqrt{9-x^2}$ is $[-3,3]$ so

$$\boxed{x = 3\sin(\theta)}$$

$$dx = 3\cos\theta$$



$$\sqrt{9-x^2} \;\updownarrow\; \sqrt{a^2-x^2}$$

$$\int_{-\pi/2}^{\pi/2} \underbrace{\sqrt{9-9\sin^2\theta}}_{\sqrt{9(1-\sin^2\theta)} \,=\, \sqrt{9\cos^2\theta}} \cdot 3\cos\theta\, d\theta = \int_{-\frac{\pi}{2}}^{\pi/2} 3\cos\theta \cdot 3\cos\theta\, d\theta \;\boxed{= 9\int_{-\frac{\pi}{2}}^{\pi/2} \cos^2\theta\, d\theta}$$

$$\boxed{\begin{aligned} x=-3 &\Rightarrow -3 = 3\sin\theta \\ -1 &= \sin\theta \\ \sin^{-1}(-1) &= \theta = -\tfrac{\pi}{2} \end{aligned}}$$

$$x = 3 = 3\sin\theta$$
$$1 = \sin\theta$$
$$\sin^{-1}(1) = \theta = \tfrac{\pi}{2}$$

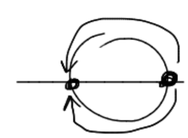
what angle gives y-coord $= -1$

$$9\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1+\cos(2\theta)}{2} = \frac{9}{2}\left[\int_{-\pi/2}^{\pi/2} d\theta + \int \cos(2\theta)\, d\theta\right]$$

$$\int \cos(2\theta)\,d\theta = \int \cos(u)\tfrac{1}{2}\,du$$

$u = 2\theta$

$du = 2\,d\theta$



$$= \frac{9}{2}\Big[\theta + \underbrace{\tfrac{1}{2}\int \cos(u)\,du}_{=\,0}\Big]$$

$$= \frac{9}{2}\Big[\theta + \overbrace{\tfrac{1}{2}\sin(2\theta)}\Big]\Bigg|_{-\frac{\pi}{2}}^{\pi/2} = \frac{9}{2}\underbrace{\Big[\tfrac{\pi}{2} + 0 - \underbrace{\big[-\tfrac{\pi}{2} + 0\big]}_{\frac{\pi}{2}+\frac{\pi}{2}}\Big]}_{\pi}$$

$$= \frac{9}{2}(\pi) = \frac{9\pi}{2}$$

Thursday – Week 3 1pm section

#13

$$\int \frac{\cos\left(\frac{\pi}{x^4}\right)}{x^5}\,dx$$

$u = \frac{\pi}{x^4} = \pi x^{-4}$ | $du = -4\pi x^{-5}dx$, $\frac{-x^5}{4\pi} \cdot du = dx$

$$= \int \frac{\cos(u)}{\cancel{x^5}} \cdot \left(\frac{-\cancel{x^5}}{4\pi}\right) du = \frac{-1}{4\pi}\int \cos(u)\,du = \boxed{-\frac{1}{4\pi}\sin\left(\frac{\pi}{x^4}\right)+C}$$

$$4\int_{-1}^{2} \frac{x}{\sqrt{x+4}}\,dx$$

$u = x+4$, $du = dx$, $x = u-4$ (from)

$4\int \frac{u-4}{\sqrt{u}}\,du$ ← easier

$= \int \boxed{\frac{u}{\sqrt{u}}} - \frac{4}{\sqrt{u}}\,du = 4\left[\int u^{1/2}du - 4\int u^{-1/2}du\right]$

$x=-1 \Rightarrow u=3$

$x=2 \Rightarrow u=6$

$$4\left[\int_3^6 u^{1/2}du - 4\int_3^6 u^{-1/2}du\right] = 4\left[\frac{2}{3}u^{\frac{3}{2}} - 4\cdot\frac{2}{1}u^{\frac{1}{2}}\right]\Bigg|_3^6 = 4\left(\frac{2}{3}(x+4)^{\frac{3}{2}} - 8(x+4)^{\frac{1}{2}}\right)\Bigg|_{-1}^{2}$$

$$4\left[\frac{2}{3}(6)^{\frac{3}{2}} - 8(6)^{1/2} - \left(\frac{2}{3}(3)^{\frac{3}{2}} - 8(3)^{1/2}\right)\right]$$

#15/

$$\int \frac{\sqrt{x^2-4}}{x}\,dx$$

$u = x^2 - 4 \Rightarrow x^2 = u+4$

$du = 2x\,dx$

$\frac{1}{2x}\,du = dx$

$$\int \frac{\sqrt{u}}{x}\cdot\frac{1}{2x}\,du \qquad \frac{1}{2}\int\frac{\sqrt{u}}{x^2}\,du$$

$$= \frac{1}{2}\int\frac{\sqrt{u}}{u+4}\,du$$ — stuck —

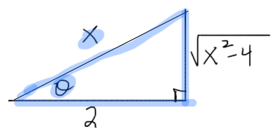
Start over

$$\boxed{\int\frac{\sqrt{x^2-4}}{x}\,dx}$$

$x = 2\sec\theta$ $\sec\theta = \frac{x}{2}$

$dx = 4\sec\theta\tan\theta$

$$= \int \sin\theta \cdot 4\sec\theta\tan\theta\,d\theta$$



$$= 4\int \tan^2\theta\,d\theta$$

$$= 4\int \sec^2\theta - 1\,d\theta$$

since $\sec\theta = \frac{x}{2}$

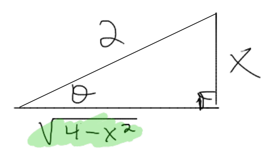
$\theta = \sec^{-1}\left(\frac{x}{2}\right)$

$$= 4(\tan\theta - \theta) + C = 4\left(\tan\left(\sec^{-1}\left(\frac{x}{2}\right)\right) + \sec^{-1}\left(\frac{x}{2}\right)\right) + C$$

what if ...

$x = 2\sin\theta \Rightarrow \frac{x}{2} = \sin\theta$

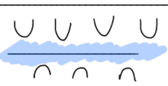
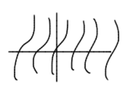
$dx = 2\cos\theta$



$y^2 + x^2 = 2^2$

$y^2 = 2^2 - x^2$

$y = \sqrt{4-x^2}$

$\sqrt{x^2-a^2}$	$\sqrt{x^2+a^2}$	$\sqrt{a^2-x^2}$
Domain $(-\infty,-a] \cup [a,\infty)$	Domain: $\mathbb{R}$	Domain: $[-a,a]$
Range $\sec(\theta)$ 	Range $\tan(\theta)$ 	Range $\sin\theta$ 

you try

$$\int \frac{1}{\sqrt{4x^2+1}}\,dx = \int \frac{\text{adj}}{\text{hyp}}\,dx = \int \cos\theta \cdot \frac{1}{2}\sec^2\theta = \frac{1}{2}\int \sec\theta$$

same

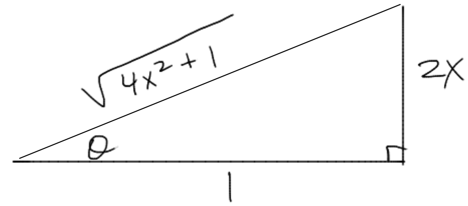
Hint: Make $\sqrt{x^2+a^2}$ appear for some choice of a.

$$x = \frac{1}{2}\tan\theta \qquad dx = \frac{1}{2}\sec^2\theta$$

$$= \int \frac{1}{\sqrt{4x^2 + \frac{4}{4}}}\,dx = \int \frac{1}{\sqrt{4\left(x^2+\frac{1}{4}\right)}}\,dx$$

$$= \int \frac{1}{2\sqrt{x^2 + 1/4}}$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{x^2 + \left(\frac{1}{2}\right)^2}}\,dx$$



$$\frac{2x}{1} = \tan\theta$$

$$x = \frac{1}{2}\tan\theta$$

$$dx = \frac{1}{2}\sec^2\theta$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\frac{1}{4}\tan^2\theta + \frac{1}{4}}}\,dx = \frac{1}{2}\int \frac{1}{\frac{1}{2}\sqrt{\tan^2\theta+1}}\,\frac{1}{2}\sec^2\theta$$

$$= \frac{1}{2}\int \frac{\sec^2\theta}{\sec\theta}\,d\theta = \frac{1}{2}\int \sec\theta\,d\theta$$

$$\theta = \tan^{-1}(2x) \qquad = \frac{1}{2}\ln\left|\sec\theta + \overbrace{\tan\theta}^{2x}\right|$$