MA161 - wk 3 — Th

warm-up

① $\int \frac{26x}{\sqrt{x^2-4}}\,dx$ $u = x^2 - 4$

Goal:
compute

$$\int \frac{1}{x^2+7x+12}\,dx$$

$$\parallel$$

$$\int \frac{1}{x+3} + \frac{-1}{x+4}\,dx$$

$$\boxed{\ln|x+3| - \ln|x+4| + C}$$

Note: The step 2 above works whenever you see DISTINCT DEGREE ONE roots downstairs

Method of Partial Fractions (Integration Technique)

General Principle:

$$\frac{1}{x^2+7x+12} = \text{sum of simpler rational expressions}$$

① Factor denom: $x^2+7x+12 = (x+3)(x+4)$

② $$\frac{1}{x^2+7x+12} = \frac{A}{x+3} + \frac{B}{x+4}$$ A, B are both unknown

③ solve ~~for~~ A, B.
"clear denominators" (multiply both sides by LCD)
$(x+3)(x+4)$

$$1 = A(x+4) + B(x+3)$$

④ continue to solve for A, B

sub: $x = -4 \Rightarrow 1 = \underbrace{A(-4+4)}_{0} + B(-4+3)$ So: $1 = -B$

$\boxed{B = -1}$

$x = -3 \Rightarrow 1 = A(-3+4) = A(1)$

$\boxed{A = 1}$

Non-distinct Roots
(repeated)

$$\int \frac{1}{x(x-1)^2}\,dx = \int \frac{1}{x} - \frac{1}{x-1} + \frac{1}{(x-1)^2}\,dx = \boxed{\ln|x| - \ln|x-1| - \frac{1}{x-1} + C}$$

① Denom: $x(x-1)(x-1)$

$$\frac{1}{x(x-1)^2} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$$

include a fraction for each sub-factor

$$\int \frac{1}{(x-1)^2}\,dx \qquad u = x-1, \quad du = dx$$

$$\int \frac{1}{u^2}\,du = \int u^{-2}\,du = \frac{u^{-1}}{-1} = \frac{-1}{(x-1)}$$

② clear denom: mult. by LCD $x(x-1)^2$

$$1 = A(x-1)^2 + B(x)(x-1) + C(x)$$

$$A(x^2-2x+1) + Bx^2 - Bx + Cx$$

$$\begin{bmatrix} Ax^2 - 2Ax + A \\ +Bx^2 - Bx + Cx \end{bmatrix} \rightarrow (A+B)x^2 + (-2A-B+C)x + A + Cx$$

$$0x^2 + 0x + 1$$

③ LHS: x^2 coef $= 0$

constant $= 1$

RHS: x^2 coef $= A + B$

constant $= A$

so $\boxed{A + B = 0}$

& $\boxed{A = 1}$

$\Rightarrow A = 1$, $B = -1$

To find C:
set $x = 1 \Rightarrow 1 = A\cdot 0 + B\cdot 1\cdot 0 + C(1)$

$C = 1$

Irreducible Case

$$-\int \frac{x}{u}\frac{1}{2x}dx = -\frac{1}{2}\int\frac{1}{u}$$

$$\int \frac{1}{x(x^2+1)}dx = \int \frac{1}{x} - \frac{x}{x^2+1}dx = \boxed{\ln|x| - \frac{1}{2}\ln|x^2+1| + C}$$

see: x^2+1 doesn't factor (irreducible) → negative discriminant

verify: Quad. Frm $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ discriminant

eg. $x^2+1 = ax^2+bx+c$

$a=1,\ c=1$

$b^2-4ac = 0-4\cdot1\cdot1 = -4 < 0$

① $\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

as usual for degree one factors, but $Bx+C$ over the irreducible factor.

② $1 = A(x^2+1) + (Bx+C)x$

set $x=0 \Rightarrow 1 = A(0^2+1) + 0 \quad A=1$

update $1 = x^2+1+Bx^2+Cx = (1+B)x^2 + Cx + 1$

$C=0$ since LHS has no x-term

$1+B=0$ " " " " x^2-term $\quad B=-1$