$$\int \frac{1}{\sqrt{1-x^2}}\,dx$$

(6) $x = \cos(\theta)$

$dx = -\sin\theta \, d\theta$

$$\int \frac{1}{\sqrt{\underbrace{1-\cos^2\theta}_{\sin^2\theta}}}(-\sin\theta \, d\theta) = \int \frac{1}{\sqrt{\sin^2\theta}} \cdot (-\sin\theta)\, d\theta = \int -d\theta$$

$$= -\theta + C$$

recall $x = \cos\theta$

$$= \boxed{-\cos^{-1}(x) + C}$$

---

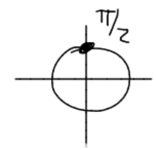
see: $\sin^{-1}(x) = -\cos^{-1}(x) + C$

Recall: $\sin(A+B) = \sin A \cos B + \sin B \cos A$

$A = \frac{\pi}{2}$, $B = -x$

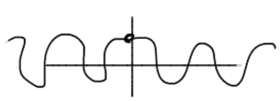
$$\sin\left(\frac{\pi}{2} - x\right) = \underbrace{\sin\frac{\pi}{2}}_{=1} \cdot \underbrace{\cos(-x)}_{\cos(x)} + \underbrace{\sin(-x)}_{-\sin(x)} \cdot \underbrace{\cos\left(\frac{\pi}{2}\right)}_{0}$$

odd



$\cos \leftrightarrow x$

$\sin \leftrightarrow y$



$\cos(x)$ is even / $\cos(-x) = \cos(x)$

set

So: $\sin\left(\frac{\pi}{2} - x\right) = \cos(x) = y$

isolate x

$\sin\left(\frac{\pi}{2} - x\right) = y$

$\frac{\pi}{2} - x = \sin^{-1}(y)$

$-\sin^{-1}(y) + \frac{\pi}{2} = x$

and if

$\cos(x) = y$

$x = \cos^{-1}(y)$

combine

$-\sin^{-1}(y) + \frac{\pi}{2} = \cos^{-1}(y)$

$\sin^{-1}(y) - \frac{\pi}{2} = -\cos^{-1}(y)$

15 w/w

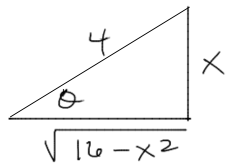
Caution!

$$\int \frac{\sqrt{x^2-16}}{x}\,dx$$

Domain $\Rightarrow \mathbb{R}$

$x = 4\sin\theta$ Range $\Rightarrow [-4,4]$

don't match

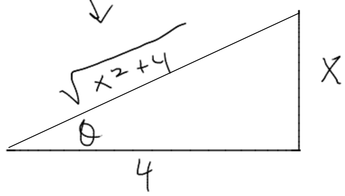


— DEAD END —

$$\int \frac{\sqrt{x^2-16}}{x}\,dx$$

Think $\sqrt{x^2-16}$ can't have $[-4,4]$ ~~has domain $\mathbb{R}$~~

no match



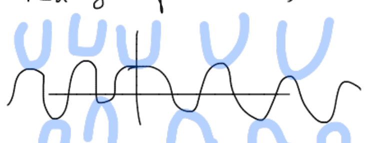
$\Rightarrow$ $x = 4\cdot\tan\theta$

b/c this can represent anything in $\mathbb{R}$

$$\int \boxed{\frac{\sqrt{x^2-16}}{x}}\,dx$$

Domain: $\sqrt{x^2-16}$ $(-\infty,-4] \cup [4,\infty)$

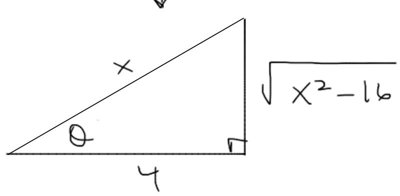
Range of $\sec^{-1}(x)$ $(-\infty,-1] \cup [1,\infty)$



wrt θ

sinθ

match



$$\boxed{\begin{aligned} &x = 4\sec\theta \quad (*) \\ &x^2 = 16\sec^2\theta \\ &dx = 4\sec\theta\tan\theta\,d\theta \end{aligned}}$$

$\sqrt{\sec^2\theta - 1} = \sqrt{\tan^2\theta} = \tan\theta$

$$\int \frac{\sqrt{16\sec^2\theta - 16}}{4\sec\theta}\cdot 4\sec\theta\tan\theta\,d\theta = 4\int \tan^2\theta\,d\theta$$

$$= 4\int \sec^2\theta - 1\,d\theta = 4\int \sec^2\theta\,d\theta - 4\int 1\,d\theta$$

$$= 4\tan\theta - 4\theta + C$$

$$= \boxed{\sqrt{x^2-16} - 4\sec^{-1}\left(\frac{x}{4}\right) + C}$$

b/c $x = 4\sec(\theta)$

Compute without calculus

$$\int_{-3}^{3}\sqrt{9-x^2}\,dx$$

$$= \tfrac{1}{2}\pi(3)^2$$

$$= \boxed{\frac{9\pi}{2}}$$

verify

w/ $x = 3\sin\theta$

$$y = \sqrt{9-x^2}$$

$$y^2 = 9 - x^2$$

$$\boxed{x^2 + y^2 = 9}$$

circle w/ radius = 3