MA161 wk 3

Today: Trig Sub
Tomorrow: Partial Fractions

When to use trig sub & which one?

Integral contains	use this	trick to remember
• $(x^2 - a^2)^{m/2}$	$x = a\sec\theta$	domain integrand $\leftrightarrow$ range trig function
• $(a^2 - x^2)^{m/2}$	$x = a\sin\theta$	
• $x^2 + a^2$ ↳ $\mathbb{R}$ domain	$x = a\tan\theta$ ↳ unlimited range	

Ex.

$$\int \frac{1}{x^2\sqrt{x^2-49}}\,dx$$

$$= \int \frac{7\sec\theta\tan\theta}{49\sec^2\theta \cdot 7\tan\theta}\,d\theta$$

$$= \frac{1}{49}\int \frac{1}{\sec\theta}\,d\theta$$

$$= \frac{1}{49}\int \cos\theta\,d\theta = \frac{1}{49}\sin\theta + C$$

$$= \frac{1}{49}\left(\frac{\sqrt{x^2-49}}{x}\right) + C$$

see: $x^2 - a^2$ think

$x = 7\sec\theta$

$dx = 7\sec\theta\tan\theta\,d\theta$

prep. your sub

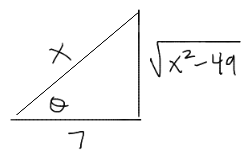
$x^2 = 49\sec^2\theta$

$\sqrt{x^2-49} = \sqrt{49\sec^2\theta - 49} = \sqrt{49(\underbrace{\sec^2\theta - 1}_{\tan^2\theta})} = 7\tan\theta$

get x back

make a △ mirroring your sub | SOHCAHTOA

$x = 7\sec\theta \Longrightarrow \frac{x}{7} = \sec\theta = \frac{1}{\cos\theta}$



$\cos\theta = \frac{7}{x} = \frac{adj}{hyp}$

use pythagorean: $x^2 = y^2 + 7^2$

Ex.

$$\int \frac{1}{(64+x^2)^2}\,dx$$

see: $64+x^2$ recognize the correct subsitution

think: $x = 8\tan\theta$ $\quad \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$dx = 8\cdot\sec^2\theta\, d\theta$

compute derivatives and consequences of our sub

$x^2 = 64\tan^2\theta$

$(64+x^2)^2 = (64+64\tan^2\theta)^2 = [64(1+\tan^2\theta)]^2$

$(2^6)^2 = 2^{12}$

pythagorean trig ID

$= 2^{12}(\underbrace{1+\tan^2\theta}_{\sec^2\theta})^2$

$= 2^{12}(\sec^4\theta)$

$$= \int \frac{1}{2^{12}(\sec^4\theta)}\,dx$$

$$= \int \frac{1}{2^{12}\cdot\sec^4\theta}\cdot \overset{2^3}{8}\sec^2\theta\, d\theta$$

everything gets replaced

sec = 1/cos

double angle formula

$$= \frac{1}{2^9}\int \frac{1}{\sec^2\theta}\,d\theta = \frac{1}{2^9}\int \cos^2\theta\, d\theta = \frac{1}{2^9}\int \frac{1+\cos(2\theta)}{2}\,d\theta$$

$\left(\frac{1}{\sec}\right)^2 = \cos^2$

integral distributes over +

$u = 2\theta$

$$= \frac{1}{2^{10}}\int 1+\cos(2\theta)\,d\theta = \frac{1}{2^{10}}\left[\int 1\,d\theta + \underbrace{\frac{1}{2}\int \cos(2\theta)\underbrace{2d\theta}_{d\theta}}_{\frac{1}{2}\int\cos(u)\,du}\right]$$

$\frac{1}{2}\sin(u)$

$\frac{1}{2}\sin(2\theta)$

u-sub

$$= \frac{1}{2^{10}}\left[\theta + \frac{1}{2}\sin(2\theta)\right] + C$$

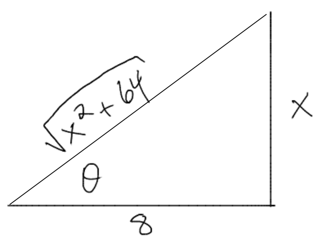
get out x back

use 1/2 angle formula

$$= \frac{1}{2^{10}}\left[\tan^{-1}\left(\frac{x}{8}\right) + \frac{1}{2}\cdot 2\sin\theta\cos\theta\right] + C$$

$\sin\theta = \frac{x}{\sqrt{x^2+64}}$ $\quad \cos\theta = \frac{8}{\sqrt{x^2+64}}$

$$= \boxed{\frac{\tan^{-1}\left(\frac{x}{8}\right)}{2^{10}} + \frac{2^{-7}x}{x^2+64} + C}$$



$\frac{x}{8} = \tan\theta = \frac{\text{opp}}{\text{adj}}$

7-3-1 #1

Ex

$$\int \frac{30x}{\sqrt{x^2-4}}\,dx$$

$$=$$

$$30\int \frac{x}{\sqrt{u}}\,\frac{1}{2x}\,du$$

$$15\int u^{-1/2}\,du$$

$$2\cdot 15\,u^{1/2}+c$$

$$\boxed{30\sqrt{x^2-4}+c}$$

see: $\sqrt{x^2-4} \rightsquigarrow x = 2\sec\theta$

↑ this does work!

or

see u-sub!

$u = x^2-4$

$du = 2x\,dx$

$\frac{1}{2x}\,du = dx$