$$\int \tan^{13}(x)\sec^4(x)\,dx$$

odd tan: $= \int \tan^{12}(x)\,\underset{u}{\sec(x)} \cdot \underbrace{\sec(x)\tan(x)\,dx}_{du}$

transform into sec

$= \int (\tan^2(x))^6$

$= (\sec^2 x - 1)^6$

expand ....

since we also have even sec(x) power
— choose lowest exponent

even sec(x):

$$\int \tan^{13}(x) \cdot \sec^2(x)\, \underbrace{\sec^2(x)\,dx}_{du}$$

transform into $\tan^2(x)$

$\Rightarrow u = \tan(x)$

$du = \sec^2(x)\,dx$

$\sec^2(x) = 1 + \tan^2 x = 1 + u^2$

$$= \int u^{13}(1+u^2)\,du$$

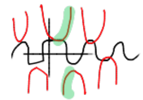
$$= \int u^{13} + u^{15}\,du = \frac{u^{14}}{14} + \frac{u^{16}}{16} + C$$

$$= \frac{\tan^{14}(x)}{14} + \frac{\tan^{16}(x)}{16} + C$$

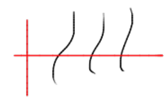
TRIG SUB

Integrand:

trick to remember: domain / range

- $(x^2-a^2)^{5/2}$ or any $^{m}/_{2}$ $\quad \xleftrightarrow{\text{sub}} \quad x = a\sec(x)$ 

- $(a^2-x^2)^{1/2}$ $\quad \xleftrightarrow{\text{sub}} \quad x = a\sin(x)$ 

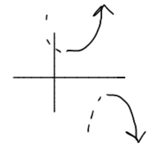
- x^2+a^2 $\quad \xleftrightarrow{\text{sub}} \quad x = a\tan(x)$ 

Ex

$$\int \frac{1}{x^2\sqrt{x^2-121}}\,dx$$

see $\sqrt{x^2-a^2}$

think: $x = a\sec\theta$



① $x = 11\sec\theta$

② $dx = 11\sec\theta\tan\theta\,d\theta$

③ $x^2 = 121\sec^2\theta$

$\sqrt{x^2-121} = \sqrt{121\sec^2\theta - 121} = \sqrt{121(\underbrace{\sec^2\theta - 1}_{\tan^2\theta})} = 11\tan\theta$

$$= \int \frac{11\sec\theta\tan\theta}{121\sec^2\theta \cdot 11\tan\theta}\,d\theta$$

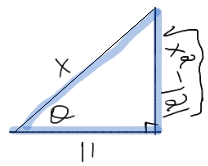
$$= \frac{1}{121}\int \frac{1}{\sec\theta}\,d\theta = \frac{1}{121}\int \cos\theta\,d\theta = \frac{1}{121}\sin\theta + C = \boxed{\frac{1}{121}\cdot\frac{\sqrt{x^2-121}}{x} + C}$$

get x back

make a △ that

mirrors your substitution

& use pythagorean



$\frac{x}{11} = \sec\theta = \frac{1}{\cos\theta}$

$\frac{11}{x} = \cos\theta$

SOH CAH TOA

$x^2 = y^2 + 11^2$

$x^2 - 121 = y^2$

$\sqrt{x^2-121} = y$

Ex

$$\int \frac{1}{(2+x^2)^2}\,dx$$

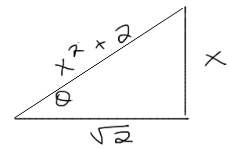
see: $2+x^2$

① $x=\sqrt{2}\tan\theta$

② $dx=\sqrt{2}\sec^2\theta\,d\theta$

③ $x^2=2\tan^2\theta$

④



$\tan\theta = \frac{x}{\sqrt{2}} = \frac{opp}{adj}$

$y^2 = x^2 + \sqrt{2}^2$

$y = x^2+2$

$$= \int \frac{1}{(\underbrace{2+2\tan^2\theta}_{})^2}\sqrt{2}\sec^2\theta\,d\theta$$

$[2(1+\tan^2\theta)]^2$

$4\cdot[\sec^2\theta]^2$

$4\sec^4\theta$

$\frac{1}{\sec^2} = \left(\frac{1}{\sec}\right)^2 = \cos^2$

$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$

$\cos^2\theta \quad \frac{1+\cos(2\theta)}{2}$

$$= \frac{\sqrt{2}}{4}\int\frac{\sec^2\theta}{\sec^4\theta}\,d\theta = \frac{\sqrt{2}}{4}\int\frac{1}{\sec^2\theta}\,d\theta = \frac{\sqrt{2}}{4}\int\cos^2\theta\,d\theta = \frac{\sqrt{2}}{4}\int\frac{1+\cos(2\theta)}{2}$$

$$= \frac{\sqrt{2}}{4}\int\frac{1}{2}+\frac{1}{2}\cos(2\theta)\,d\theta = \frac{\sqrt{2}}{4}\left[\frac{1}{2}\theta + \underbrace{\frac{1}{2}\frac{1}{2}\int\cos(2\theta)\,2\,d\theta}_{}\right] =$$

$\frac{1}{4}\int\cos(u)\,du = \frac{1}{4}\sin(u) = \frac{1}{4}\sin(2\theta)$

$\quad = \frac{1}{4}(2\sin\theta\cos\theta)$

$\frac{1}{4}\,\frac{x}{\sqrt{x^2+4}}\cdot\frac{\sqrt{2}}{\sqrt{x^2+4}}$

Get x back

$\theta = \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$

$\frac{x}{\sqrt{2}} = \tan\theta$