. . . work to do even with a "cheat sheet" . . . .

1. Use #32. $u = x \Longrightarrow du = dx$, $a = \sqrt{3}$

$$\int \frac{\sqrt{3-x^2}}{x}\,dx = \int \frac{\sqrt{\left(\sqrt{3}\right)^2 - u^2}}{u}\,du$$

$$= \sqrt{3-x^2} - \sqrt{3}\ln\left|\frac{\sqrt{3}+\sqrt{3-x^2}}{\sqrt{3}}\right| + C$$

2. Use #101. $u = x \Longrightarrow du = dx$

$$\int x^3 \ln x\,dx = \frac{x^{3+1}}{(3+1)^2}\left[(3+1)\ln x - 1\right] + C$$

$$= \frac{x^4}{16}\left[4\ln x - 1\right] + C = \frac{x^4}{4}\ln x - \frac{x^4}{16} + C$$

3. First replace $\sqrt{x}$. Let $w = \sqrt{x} \Longrightarrow dw = \frac{1}{2}\left(\frac{1}{\sqrt{x}}\right)dx$
$\Longrightarrow 2wdw = dx$

$$\int \cos\sqrt{x}\,dx = \int 2w\cos w\,dw = 2\int w]cosw\,dw$$

Now use #83

$$2\int w\cos w\,dw = 2\left[\cos w + w\sin w\right] + C = 2\cos\sqrt{x} + 2\sqrt{x}\cos\sqrt{x} + C$$

4. First replace x^2. Let $w = 4x^2 \Longrightarrow dw = 8xdx \Longrightarrow \frac{1}{8}dw = xdx$

$$\int \frac{x}{16x^4 - 1}\,dx = \int \frac{1}{8}\left(\frac{1}{w^2-1}\right)dw = \frac{1}{8}\int \frac{1}{w^2-1}\,dw$$

Now use #20, with $a = 1$ and $u = w$

$$\frac{1}{8}\int \frac{1}{w^2-1}\,dw = \frac{1}{8}\left[\frac{1}{2(1)}\ln\left|\frac{w-1}{w+1}\right|\right] + C = \frac{1}{16}\ln\left|\frac{4x^2-1}{4x^2+1}\right| + C$$

5. Use #113 with $9x^2 = u^2$. Let $u = 3x \Longrightarrow \frac{1}{3}du = dx$ and $2au = 2\left(\frac{5}{6}\right)(3x)$, so $a = \frac{5}{6}$.

$$\int \sqrt{5x - 9x^2}\, dx = \int \sqrt{2\left(\frac{5}{6}\right)(3x) - (3x)^2}\, dx$$

$$= \frac{1}{3}\left[\frac{3x - \frac{5}{6}}{2}\sqrt{2\left(\frac{5}{6}\right)(3x) - (3x)^2} + \frac{\left(\frac{5}{6}\right)^2}{2}\cos^{-1}\left(\frac{\frac{5}{6} - 3x}{\frac{5}{6}}\right)\right] + C$$

6. First replace $\cos 3x$. Let $w = \cos 3x \Longrightarrow dw = -3\sin 3x dx$. So $-\frac{1}{3}dw = \sin 3dx$.

$$\int \frac{\sin 3x}{(\cos 3x)(\cos 3x + 1)}\, dx = -\frac{1}{3}\int \frac{1}{w(w+1)}\, dw$$

Now use #49, with $a = b = 1$.

$$-\frac{1}{3}\left[\frac{1}{1}\ln\left|\frac{w}{1+w}\right|\right] + C = -\frac{1}{3}\ln\left|\frac{\cos 3x}{1 + \cos 3x}\right| + C$$