

Integration Techniques Quiz

1. use $u = \sin \pi x \Rightarrow du = \pi \cos \pi x dx$

$$\int \sin^2 \pi x \cos^5 \pi x dx = \int \frac{1}{\pi} u^2 (1 - u^2)^2 du = \frac{1}{\pi} \int u^2 - 2u^4 + u^6 du$$
$$\frac{1}{3\pi} \sin^3 \pi x - \frac{2}{5\pi} \sin^5 \pi x + \frac{1}{7\pi} \sin^7 \pi x + C$$

2. use $w = 2x + 1$ (You don't have to, but the problem turns out a lot cleaner this way)

$$\int \ln(2x + 1) dx = \int \frac{1}{2} \ln w dw$$
$$u = \ln w \quad dv = \frac{1}{2} dw$$
$$du = \frac{1}{w} dw \quad v = \frac{1}{2} w$$
$$\int \frac{1}{2} \ln w dw = \frac{1}{2} w \ln w - \int \frac{1}{w} \cdot \frac{1}{2} w dw = \frac{1}{2} w \ln w - \int \frac{1}{2} dw$$
$$= \frac{1}{2} w \ln w - \frac{1}{2} w + C$$
$$= \frac{1}{2} (2x + 1) \ln(2x + 1) - \frac{1}{2} (2x + 1) + C$$

3. use $x = 4 \tan \theta \rightarrow dx = 4 \sec^2 \theta d\theta$

$$\int \frac{1}{\sqrt{x^2 + 16}} dx = \int \frac{1}{\sqrt{16 \sec^2 \theta}} (4 \sec^2 \theta) d\theta$$
$$\int \sec \theta d\theta = \ln |\sec \theta + \tan \theta| + C$$
$$= \ln \left| \frac{\sqrt{x^2 + 16}}{4} + \frac{x}{4} \right| + C$$

$$\int \frac{1}{(x+5)^2(x-1)} dx$$

work to pull rational function apart

$$\frac{1}{(x+5)^2(x-1)} = \frac{A}{x-1} + \frac{B}{x+5} + \frac{C}{(x+5)^2}$$

$$1 = A(x+5)^2 + B(x+5)(x-1) + C(x-1)$$

$$1 = (A+B)x^2 + (10A+4B+C)x + 25A - 5B - C$$

$$\text{when } x = 1 \implies 1 = 36A \implies A = \frac{1}{36} \text{ and } B = -\frac{1}{36}$$

$$\text{when } x = -5 \implies 1 = 4C \implies \frac{1}{4}$$

$$\begin{aligned} \int \frac{1}{(x+5)^2(x-1)} dx &= \int \frac{1}{36} \cdot \frac{1}{x-1} - \frac{1}{36} \cdot \frac{1}{x+5} + \frac{1}{4} \cdot \frac{1}{(x+5)^2} dx \\ &= \frac{1}{36} \ln|x-1| - \frac{1}{36} \ln|x+5| - \frac{1}{4(x+5)} + C \end{aligned}$$