MA163 wk 4 Fri   7.7

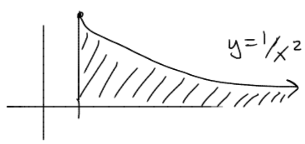
Improper Integrals eg., $\int_a^\infty f(x)\,dx$ ← similar to: $\int_{-\infty}^a f(x)\,dx$ $\int_{-\infty}^\infty f(x)\,dx$

$$\boxed{\text{FTC: } \int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b)-F(a) \quad \text{definite int}}$$

$$\int_a^\infty f(x)\,dx \overset{def}{=} \lim_{R\to\infty}\int_a^R f(x)\,dx$$

Idea:

$$\int_1^\infty \frac{1}{x^2}\,dx = \lim_{R\to\infty}\int_1^R \underbrace{\frac{1}{x^2}}_{=x^{-2}}\,dx = \lim_{R\to\infty} \frac{x^{-1}}{-1}\Bigg|_1^R = \lim_{R\to\infty} \frac{R^{-1}}{-1} - \frac{1^{-1}}{-1} = \lim_{R\to\infty} \underbrace{\frac{-1}{R}}_{\to 0} + 1 = 1$$



$$\int_{-\infty}^\infty f(x) \overset{def}{=} \int_{-\infty}^a f(x)\,dx + \int_a^\infty f(x)\,dx$$ ← these are computed as

$$\int_1^\infty \frac{1}{x}\,dx = \lim_{R\to\infty}\int_1^R \frac{1}{x}\,dx = \lim_{R\to\infty} \ln|x|\Big|_1^R = \lim_{R\to\infty} \ln(R) - \ln(1) = \ln(\infty) - \ln(1)$$

$$= \infty - 0$$

$$= \infty$$

drop abs val since $1 > 0$, $R > 0$

$$\int_1^\infty \frac{1}{\sqrt{x}}\,dx = \lim_{R\to\infty}\int_1^\infty x^{-1/2} = 2x^{1/2}\Big|_1^R = \lim_{R\to\infty} 2\left(\sqrt{R} - 1^{1/2}\right) = 2\sqrt{\infty} - 1 = \infty$$

we've seen:

p	$\int_0^\infty \frac{1}{x^p}dx$
$1/2$	∞
1	∞
2	finite $(=1)$
3	finite $(=1/2)$

$$\int_1^\infty \frac{1}{x^3}\,dx = \int x^{-3}\,dx = \frac{x^{-2}}{-2}\Big|_1^\infty = -\frac{1}{2}\cdot\frac{1}{x^2}\Big|_1^R = -\frac{1}{2}\left(\frac{1}{R^2} - 1\right)\Bigg|_{R\to\infty} = -\frac{1}{2}\left(\overbrace{\frac{1}{\infty^2}}^{\to 0} - 1\right)$$

$$= \frac{1}{2}$$

Note: Initial Point doesn't affect ∞ vs finite dichotomy.

$1000 \cdot 1000$

Ex Know $\int_1^\infty \frac{1}{\sqrt{x}}\,dx = \infty$, what about $\int_{1,000,000}^\infty \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x}\Big|_{1,000,000}^R = 2\left(\sqrt{R} - \sqrt{1,000,000}\right)$

$$= 2\left(\sqrt{\infty} - 1000\right)$$

$$= \infty$$

$p = -4$ $\quad (-4 < 1 \Rightarrow$ excepect $\int \frac{1}{x^{-4}} dx =$

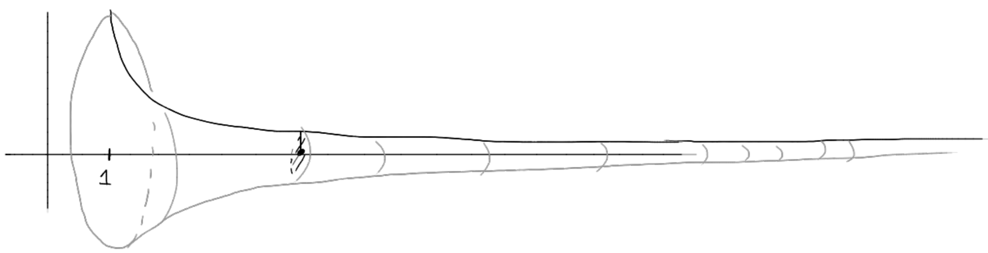
$$\int_1^\infty \frac{1}{x^{-4}} dx = \int_1^\infty x^4 dx = \frac{x^5}{5}\Big|_1^\infty = \frac{\infty^5}{5} - \frac{1}{5} = \infty$$

Convergence / Divergence
of
p-interals

$$\int_1^\infty \frac{1}{x^p} dx = \begin{cases} \text{finite (converges)} & \text{if } p > 1 \\ \infty \text{ diverges} & \text{if } p \le 1 \end{cases}$$

Gabrielle's Horn

Revolve $y = 1/x$ about x-axis



Vol. of Gab's Horn: $= \int_1^\infty \pi \cdot \frac{1}{x^2} dx = \pi$

Finite Volume means, you could think of the horn as a bucket and fill it with ONLY a finite amount of pain

πr^2

$r = \frac{1}{x}$

$= A = \pi\left(\frac{1}{x^2}\right)$

Surface Area of Gab's Horn: = circumference of a slice × arc length of curve, integrated

for now use calc.

$S = \int_1^\infty \underbrace{2\pi\left(\frac{1}{x}\right)}_{C = 2\pi r} \cdot \sqrt{1 + \frac{1}{x^2}}\, dx = \infty$

yet, it takes an infinite amount of paint to cover the outside`