LIPET

exp.

power

Exam 1 Guide

1.

$$\int x^4 e^{3x}\,dx = \frac{x^4}{3}e^{3x} - \frac{4x^3}{9}e^{3x} + \frac{12x^2}{27}e^{3x} - \frac{24}{81}e^{3x} + \frac{24}{243}e^{x} + C$$

Multiple I.B.P.'s Technique

alternate sign	differentiate u	integrate dv
$+$	x^4	e^{3x}
$-$	$4x^3$	$\frac{1}{3}e^{3x}$
$+$	$12x^2$	$\frac{1}{9}e^{3x}$
$-$	$24x$	$\frac{1}{27}e^{3x}$
$+$	24	$\frac{1}{81}e^{3x}$
$-$	0	$\frac{1}{243}e^{3x}$

stop @ zero

2.

$$\int \frac{4x-1}{x^2-5x-14}\,dx = \int \frac{A}{x-7} + \frac{B}{x+2}\,dx$$

$(x-7)(x+2)$

hope for u-sub: numerator = multiple of derivative of denom | no u-sub

3.

$$\int x^4 \sec^2(x^5)\,dx = \frac{1}{5}\int \sec^2(u)\,5x^4\,dx = \frac{1}{5}\int \sec^2(u)$$

$u = x^5 \quad du = 5x^4\,dx$

odd $\sin(x)$!

4.
$$\int \sin^5\theta\, d\theta = \int \underbrace{\sin^4(x)}_{\text{pythag.}} \underbrace{\sin(x)\,dx}_{\text{want} = du}$$

$\sin^4 = (\sin^2 x)^2$

$$= \int (1-\cos^2(x))^2 \sin(x)\,dx$$

u-sub now

$u = \cos(x)$

$du = -\sin(x)\,dx$

$$= -\int (1-u^2)^2\,du$$

FOIL, POWER RULE, GET X-BACK

$$= -\int 1 - 2u^2 + u^4\,du = -u + \frac{2u^3}{3} - \frac{u^5}{5} + C$$

$$= -\cos(x) + \frac{2}{3}\cdot\cos^3(x) - \frac{1}{5}\cos^5(x) + C$$

5.
$$\int \frac{x^5}{\sqrt{1-4x^2}}\,dx = \int \frac{\frac{1}{32}\sin^5\theta}{\cos\theta}\cdot\frac{1}{2}\cos\theta\,d\theta = \frac{1}{64}\int \sin^5\theta\,d\theta = \frac{1}{64}(\text{ans to \#4})$$

① see $1-4x^2$ as $k(a^2-x^2)$

② factor 4 out: $4(\frac{1}{4}-x^2) = 4(a^2-x^2)$

③ think: $x = a\sin\theta$, $a = \sqrt{1/4} = 1/2$

$x = \frac{1}{2}\sin\theta$

$x = \frac{1}{2}\sin\theta$

$x^5 = \frac{1}{32}\sin^5\theta$

$dx = \frac{1}{2}\cos\theta\,d\theta$

$$\sqrt{1-4x^2} = \sqrt{4(\tfrac{1}{4}-x^2)} = 2\sqrt{\tfrac{1}{4}-(\tfrac{1}{2}\sin\theta)^2}$$
$$= 2\sqrt{\tfrac{1}{4}-\tfrac{1}{4}\sin^2\theta}$$
$$= 2\sqrt{\tfrac{1}{4}(1-\sin^2\theta)}$$
$$= \frac{2}{2}\cos\theta = \cos\theta$$

$u = \sec^{-1}x \quad dv = 2x$

6.

(a) $du = \dfrac{1}{x\sqrt{x^2-1}} \quad v = x^2 \longrightarrow$

$$\int 2x\sec^{-1}x\,dx = x^2 \cdot \sec^{-1}x - \underbrace{\int \frac{x^2}{x\sqrt{x^2-1}}\,dx}_{\text{cancel} \ \& \ u=\text{sub}}$$

(b) LIPET

LIATE

$$\int ln(\sqrt(x))\,dx = \int \ln|\sqrt{x})\,dx$$

I.B.P.

$$= x \cdot \ln(\sqrt{x}) - \underbrace{\int \frac{x}{2x}\,dx}_{\text{cancel} \ldots}$$

$u = \ln(\sqrt{x}) \quad dv = dx$

$du = \dfrac{1}{x^{1/2}} \cdot \dfrac{1}{2}x^{-1/2} \quad v = x$

$= \dfrac{1}{2x}$

(c)

$$\int \sec^3\theta\,d\theta = \sec\theta\tan\theta - \int \tan^2\theta \cdot \sec\theta\,d\theta$$

I.B.P

$\int \sec\theta \cdot \sec^2\theta\,d\theta$

$u = \sec\theta \quad dv = \sec^2\theta$

$du = \sec\theta\tan\theta \quad v = \tan\theta$

Pythag

$= -\int(\sec^2\theta - 1)\sec\theta\,d\theta$

$= -\int \sec^3\theta\,d\theta + \int \sec\theta\,d\theta$

$\int \sec\theta \dfrac{\sec + \tan}{\sec + \tan}$

$= \int \dfrac{\sec^2 + \sec\tan}{\sec + \tan}$

du, $u =$

$= \ln|\sec\theta + \tan\theta|$

(d)

$$\int e^{2x}\sin 3x\,dx =$$

(e)

$$\int \frac{x^2}{\sqrt{16-x^2}}\,dx =$$

trig. sub

$x = 4\sin\theta$

(f)

$$\int \frac{3x^2+8x+6}{(x+1)^3}\,dx = \int \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$$

repeated

LIPET
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(d) see: product $\Rightarrow$ I.B.P.

$\int e^{2x} \sin 3x \, dx = \frac{-e^{2x}}{3} \cdot \cos(3x) + \underbrace{\int \frac{1}{3}\cos(3x) \cdot 2e^{2x} \, dx}_{\text{equal in complexity to OG.}}$

$u = e^{2x}$ $\quad dv = \sin 3x$

$du = 2e^{2x}$ $\quad v = -\frac{1}{3}\cos(3x)$

$= \frac{-e^{2x}}{3} \cdot \cos(3x) + \frac{2}{9} e^{2x} \sin(3x) - \int \frac{4}{9} \sin(3x) \, e^{2x} \, dx$

$u = 2e^{2x}$ $\quad dv = \frac{1}{3}\cos(3x)$

$du = 4e^{2x}$ $\quad v = \frac{1}{9}\sin(3x)$

$\frac{13}{9} \int e^{2x} \sin(3x) \, dx = \frac{-e^{2x}}{3} \cdot \cos(3x) + \frac{2}{9} e^{2x} \sin(3x)$
$+C$

mult. by $\frac{9}{13}$ to get ans