MA163 Wk 4 — Mon

For multiple I.B.P's:

alternate sign	differentiate u	integrate dv
+	x^5	e^{2x}
−	$5x^4$	$\frac{1}{2}e^{2x}$
+	$20x^3$	$\frac{1}{4}e^{2x}$
−	$60x^2$	$\frac{1}{8}e^{2x}$
+	$120x$	$\frac{1}{16}e^{2x}$
−	120	$\frac{1}{32}e^{2x}$
+	0	$\frac{1}{64}e^{2x}$

$$\int x^5 \cdot e^{2x} =$$

$$\frac{x^5}{2}e^{2x} - \frac{5x^4}{4}e^{2x} + \frac{20x^3}{8}e^{2x} - \frac{60x^2}{16}e^{2x} + \frac{120x}{32}e^{2x} - \frac{120}{64}\cdot e^{2x} + C$$

Exam 1 Guide

1. I.B.P ! 4 times

$$\int x^4 e^{3x}\, dx =$$

2.

$$\int \frac{4x-1}{\underset{(x-7)(x+2)}{x^2-5x-14}}\, dx = \int \frac{A}{x-7} + \frac{B}{x+2}\, dx$$

- Is the numerator a mult. of derivative of denom? No

  If so, u-sub

- Partial Fraction

3.

$$\int x^4 \sec^2(x^5)\ dx =$$

$u = x^5$

$K\int \sec^2 u\, du$

4.

convert to $\cos\theta$

$$\int \sin^5 \theta \, d\theta = \int \sin^4\theta \, \sin\theta$$

odd power of $\sin\theta \Rightarrow$ $u = \cos\theta$

5.

$$\int \frac{x^5}{\sqrt{1-4x^2}} \, dx =$$

look for u-sub : NO

$\sqrt{a^2 - x^2}$ ... $x = a\sin\theta$

TRIG SUB

LIATE
LIPET

u? NO
see: product
$\Rightarrow$ I.B.P.

6.
(a) $u = \sec^{-1} x$ $\quad dv = 2x$

$du = \frac{1}{x\sqrt{x^2-1}}$ $\quad v = x^2$

$$\int 2x \sec^{-1} x \, dx =$$

(b) $\int \ln(\sqrt{x})\,dx$ $\quad u = \ln(\sqrt{x})$ $\quad dv = dx$

$du =$ $\quad v =$

$$\int ln(\sqrt{(x)})\, dx =$$

	power
sin	odd
cos	odd
sec	even
tan	odd

(c)

$u = \tan$
$du = \sec^2$

$$\int \sec^3\theta \, d\theta = \int \sec^2\theta \sec\theta \, d\theta = \int (\tan^2\theta + 1)\sec\theta\, d\theta$$

$$= \int \underbrace{\tan^2\theta \sec\theta}_{???} + \sec\theta\, d\theta \quad \text{stuck.}$$

see:
I.B.P.
twice, then
solve for
$\int e^{2x} \sin 3x$ like we did above.

(d)

$$\int e^{2x} \sin 3x \, dx =$$

$u = \sin 3x$ $\quad dv = e^{2x}$
$du = 3\cos 3x$ $\quad v = \frac{1}{2}e^x$

$= \frac{1}{2}e^x \cdot \sin 3x - \int 3\cos 3x\, e^{2x}$

$u = 3\cos 3x$ $\quad dv = e^{2x}$
$du = -9\sin 3x$ $\quad v = \frac{1}{2}e^{2x}$

$= \frac{1}{2}e^x \cdot \sin 3x - \left(\frac{3}{2}\cos 3x\, e^{2x} - \int \frac{-9}{2} e^{2x} \sin(3x)\,dx\right)$

$= \frac{1}{2}e^x \cdot \sin 3x - \frac{3}{2}\cos 3x\, e^{2x} - \frac{9}{2}\int e^{2x}\sin(3x)\,dx$

$$\boxed{\int e^{2x}\sin 3x = \frac{2}{11}\left(\frac{1}{2}e^{2x}\cdot \sin 3x - \frac{3}{2}\cos 3x\, e^{2x}\right)}$$

(e)

trig sub
$x = 4\sin x$

$$\int \frac{x^2}{\sqrt{16 - x^2}}\, dx =$$

(f)

$$\int \frac{3x^2 + 8x + 6}{(x+1)^3}\, dx =$$

repeated

$$= \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$$

$\int \sec^3\theta\, d\theta = \int \sec\theta \cdot \sec^2\theta\, d\theta$

I.B.P.

$u = \sec\theta$ $\quad dv = \sec^2\theta\, d\theta$

$du = \sec\theta\tan\theta$ $\quad v = \tan\theta$

$= \sec\theta\tan\theta - \int \tan^2\theta \sec\theta\, d\theta$

$\tan^2\theta = \sec^2\theta - 1$

$= \sec\theta\tan\theta - \int (\sec^2\theta - 1)\sec\theta\, d\theta$

$= \sec\theta\tan\theta - \int \sec^3\theta\, d\theta + \sec\theta\, d\theta$

$2\int \sec^3\theta\, d\theta = \sec\theta\tan\theta + \int \sec\theta \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}\, d\theta$

$= \int \frac{\sec^2\theta + \sec\theta\tan\theta}{\sec\theta + \tan\theta}$

$u = \sec\theta + \tan\theta$
$du = \sec^2\theta + \sec\theta\tan\theta$

$\int \frac{du}{u} = \ln|u| = \ln|\sec\theta + \tan\theta|$

$$\int \sec^3\theta\, d\theta = \frac{1}{2}\cdot\left(\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\right) + C$$

INTEGRATION TECHNIQUES

This review of integration techniques is in no way complete. It is vital for your success that you attempt a large number of problems from the text (even more than are assigned). There is no substitute for practice and experience. I hope that this guide helps you organize your studying.

On page 495 of the text you can see a table of the integrals we can do in one step. Really, the integrals from this table that I want you to assume are doable in one step are 1-14 and 17. Those are the ones you can assume. If your integral is not one of those, then you need some simplifying method. The first thing you should do is look for any possible substitutions or algebraic simplifications. Then you should try one of our four new methods. These methods, and when to choose them, are illustrated below:

YOUR INTEGRAL

Special Forms

$\int \sec x \, dx$ – mult. & ÷ by $\sec x + \tan x$

$\int \sec^3 x \, dx$ – 1. I.B.P. 2. solve for $\int \sec^3 x$

SIMPLICATION OR u-SUBSTITUTION

Products, log's, inverse trig

INTEGRATION BY PARTS

u =   dv =
du =   v =

If you're stuck on choosing *u* remember LIPET. (But after you get comfortable with this method, you shouldn't need LIPET anymore)

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sin's, cos's, tan's, sec's

TRIG. INTEGRALS

1. Odd cos → u = sin(x)
2. Odd sin → u = cos(x)
3. Even sec → u = tan(x)
4. Odd tan → u = sec(x)
5. Even sin & cos → Half Angle Identities

In the first 4 cases you need the identities:

$\sin^2(x) = 1 - \cos^2(x)$
$\cos^2(x) = 1 - \sin^2(x)$
$\tan^2(x) = \sec^2(x) - 1$
$\sec^2(x) = \tan^2(x) + 1$

For the 5th case, you need the half angle identities:

$\sin^2(x) = (1 - \cos(2x))/2$
$\cos^2(x) = (1 + \cos(2x))/2$
$\sin(x)\cos(x) = \sin(2x)/2$

$a^2 - x^2$, $x^2 + a^2$, $x^2 - a^2$
or if quadratic doesn't factor

TRIG. SUBSTITUTION

If the quadratic has a linear term ('middle term') and it doesn't factor, then you need to complete the square. (1/2 of middle term, square, add and subtract value)
The rest of the method follows by making the correct substitution.

$x = a\sin(\theta)$
$x = a\tan(\theta)$
$x = a\sec(\theta)$

At the end, draw and label the TRIANGLE to get back to x's.

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rational functions where the bottom factors

PARTIAL FRACTIONS

Divide if the power of top is bigger than power on bottom.

Then factor the bottom and set up and solve the partial fraction decomposition.

Distinct Linear Factors
→ Determine a constant for each factor.

Non-Distinct Linear Factors
→ Determine a constant for each factor, along with each power from 1 up to the number of times repeated.

Irreducible Quadratic Factor
→ Complete the square, the numerator of the factor is Ax+B.