MA163 wk 4 thur

Back

$-8 \longrightarrow \frac{80-8}{80} \times 100 \approx \boxed{90}$

curve:

$f(x) = \frac{5}{8} \cdot x + 35$

MA161 Exam 1

Show all work to receive credit. Access to internet / graphing calculator / etc during the exam will result in a score of 0.

1. see product $\Rightarrow$ think I.B.P.

$$\int x^3\cos(2x)\,dx = \frac{x^3}{2}\sin(2x) + \frac{3x^2}{4}\cos(2x) - \frac{6x}{8}\sin(2x) - \frac{6}{16}\cos(2x) + C$$

LIATE

sign	u	dv
+	x^3	$\cos(2x)$
−	$3x^2$	$\frac{1}{2}\sin(2x)$
+	$6x$	$-\frac{1}{4}\cos(2x)$
−	6	$-\frac{1}{8}\sin(2x)$
+	0	$\frac{1}{16}\cos(2x)$

2. [partial fractions]

$$\int \frac{2x+1}{(x-2)(x-6)}\,dx = \int \frac{\overset{= -5/4}{A}}{x-2} + \frac{\overset{= 13/4}{B}}{x-6}\,dx = -\frac{5}{4}\int\frac{1}{x-2}dx + \frac{13}{4}\int\frac{1}{x-6}dx$$

u = sub

$$= \boxed{-\frac{5}{4}\ln|x-2| + \frac{13}{4}\ln|x-6| + C}$$

clear: $2x+1 = A(x-6) + B(x-2)$

$x=2$ $\quad 2\cdot2+1 = 5 = A(-4) \quad A = -5/4$

$x=6$ $\quad 13 = B\cdot 4 \quad B = 13/4$

$$\int u^n\,du$$

3. u-sub

$$\frac{1}{4}\int 4x^3\sin^2(x^4)\,dx = \frac{1}{4}\int \sin^2(u)\,du = \frac{1}{4}\int\frac{1}{2}(1-\cos(2u))\,du$$

$u = x^4$

$du = 4x^3dx$

$$= \frac{1}{4}\int\frac{1}{2}\,du - \frac{1}{4}\int\cos(2u)\,du -$$

$w = 2u$

$dw = 2du$

$\frac{1}{2}dw = du$

$$= \boxed{\frac{1}{8}x^4 - \frac{1}{8}\sin(2x^4) + C}$$

$-\frac{1}{4}\int\cos(w)\frac{1}{2}dw = -\frac{1}{8}\sin(w)$

$= -\frac{1}{8}\sin(2u)$

$= -\frac{1}{8}\sin(2x^4)$

4. sep. out both secθ tanθ

$\sin^2 + \cos^2 = 1$

$$\int \tan^3\theta \sec^3\theta \, d\theta = \int \underbrace{\tan^2\theta}_{\text{pythag, get w/ } \sec^2\theta} \sec^2\theta \, \underbrace{\sec\theta \tan\theta \, d\theta}_{\text{want } du} \iff \underline{u = \sec\theta}$$

$$= \int (\sec^2\theta - 1)\sec^2\theta \, \sec\theta\tan\theta \, d\theta$$

$$= \int (u^2 - 1)\,u^2 \, du = \int u^4 - u^2 \, du \overset{\text{power}}{=} \frac{u^5}{5} - \frac{u^3}{3} + C$$

$$= \boxed{\frac{\sec^5\theta}{5} - \frac{\sec^3\theta}{3} + C}$$

5. see: $\sqrt{x^2+4}$ think $x = 2\tan\theta$

$$\int x^3\sqrt{x^2+4}\,dx =$$

$x^3 = 8\tan^3\theta$

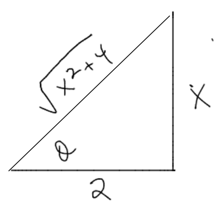
$\sqrt{x^2+4} = \sqrt{4\tan^2\theta + 4} = \sqrt{4\underbrace{(\tan^2\theta + 1)}_{\sec^2\theta}} = 2\sec\theta$

$$= \int 8\tan^3\theta \cdot 2\sec\theta \cdot 2\sec^2\theta \, d\theta$$

$dx = 2\sec^2\theta \, d\theta$

$$= 32\int \tan^3\theta \sec^3\theta \, d\theta \overset{\#4}{=} 32\left[\frac{\sec^5\theta}{5} - \frac{\sec^3\theta}{3} + C\right] = 32\left[\frac{1}{5}\left(\frac{\sqrt{x^2+4}}{2}\right)^5 - \frac{1}{3}\left(\frac{\sqrt{x^2+4}}{2}\right)^3\right] + C$$

$=$



$\frac{x}{2} = \tan\theta = \frac{\text{opp}}{\text{adj}}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{\text{opp}}{\text{hyp}}} = \frac{\text{hyp}}{\text{adj}}$

6. (you need to do at least three of the following; additional ones may be done for extra credit.)
Evaluate at least three of the following integrals:

(a)

$$\int 2x \tan^{-1} x \, dx =$$

(c)

$$\int \frac{\sqrt{x^2-9}}{x^4} \, dx =$$

collect

x2

(d) I.B.P. $e^{3x} \rightsquigarrow e^{3x}$

sin → cos →

$$\int e^{3x} \sin 4x \, dx =$$

periodic

(e)

$$\int \frac{x^2}{\sqrt{16-x^2}} \, dx =$$

(f)

$$\int \frac{x^2+4x+6}{x(x^2+2x+1)} \, dx =$$

$$\int 2x \tan^{-1}x \, dx = x^2 \tan^{-1}x - \int \frac{x^2}{1+x^2}\,dx$$

no u-sub

see: $1+x^2 \Rightarrow x = \tan\theta$

$x^2 = \tan^2\theta$

$1+x^2 = \sec^2\theta$

$dx = \sec^2\theta \, d\theta$

see product | $u = \tan^{-1}x$ $dv = 2x$

LIATE | $du = \frac{1}{1+x^2}$ $v = x^2$

$$= x^2 \tan^{-1}x - \underbrace{\int \frac{\tan^2\theta}{\sec^2\theta} \cdot \sec^2\theta \, d\theta}$$

$-\int \tan^2\theta \, d\theta$

$-\int \sec^2\theta - 1 \, d\theta$

$-\underbrace{\tan\theta}_{=x} - \underset{=\tan^{-1}x}{\theta}$

get x back

$x^2 \tan^{-1}x - x - \tan^{-1}x + C$

$$\boxed{(x^2-1)\tan^{-1} - x + C}$$

$$\int \frac{\sqrt{x^2-9}}{x^4}\,dx = \int \frac{3\tan\theta}{81\sec^4\theta}\, 3\sec\theta\tan\theta\, d\theta = \frac{1}{9}\int \frac{\tan^2\theta}{\sec^3\theta}\,d\theta$$

see $\sqrt{x^2-9}$, $x^2 > 9$

$\Rightarrow$ $x = 3\sec\theta$, $\frac{x}{3} = \sec\theta$ | $\frac{3}{x} = \cos\theta$

$x^4 = 81\sec^4\theta$

$\sqrt{x^2-9} = \sqrt{9\sec^2-9} = 3\tan\theta$

$dx = 3\sec\theta\tan\theta\,d\theta$

$$= \frac{1}{9}\int \tan^2\theta \cdot \cos^3\theta\,d\theta$$

$$= \frac{1}{9}\int \frac{\sin^2\theta}{\cos^2\theta}\cdot\cos^3\theta$$

$$= \frac{1}{9}\int \sin^2\theta\,\cos\theta\,d\theta$$

$u = \sin\theta$, $du = \cos\theta\,d\theta$

$$= \frac{1}{9}\int u^2\,du$$

$$= \frac{1}{9}\frac{u^3}{3} + c = \frac{(\sin\theta)^3}{27} + c = \frac{1}{27}\left(\frac{\sqrt{x^2-9}}{x}\right)^3$$

$$= \boxed{\frac{(x^2-9)^{3/2}}{27x^3} + c}$$