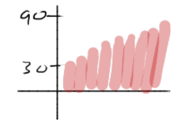
MA103 wk 4 Thur

Exam 1

$\boxed{-8}$ on Back $\longrightarrow$ $\dfrac{80-8}{80} \times 100 = \dfrac{70}{80} \times 100 = \dfrac{7}{8} \times 100 \approx 90$



Curve: $c(x) = \frac{5}{8}(x) + 35$

grade		curve
40	$\longrightarrow$	60
73	$\longrightarrow$	80

MA161 Exam 1

Show all work to receive credit. Access to internet / graphing calculator / etc during the exam will result in a score of 0.

1. I.B.P. LIPET ⇒

$$\int x^3\cos(2x)\,dx = \frac{x^3}{2}\sin(2x) + \frac{3x^2}{4}\cos(2x) - \frac{6x}{8}\sin(2x) - \frac{6}{16}\cos(2x) + C$$

sign	u	dv
+	x^3	$\cos(2x)$
−	$3x^2$	$\frac{1}{2}\sin(2x)$
+	$6x$	$-\frac{1}{4}\cos(2x)$
−	6	$-\frac{1}{8}\sin(2x)$
+	0	$\frac{1}{16}\cos(2x)$

2.

$$\int \frac{2x+1}{(x-2)(x-6)}\,dx = \int \frac{\overset{-5/4}{A}}{x-2} + \frac{\overset{13/4}{B}}{x-6}\,dx = -\frac{5}{4}\ln|x-2| - \frac{13}{4}\ln|x-6| + C$$

[Partial Fractions] u-sub

clear: $2x+1 = A(x-6) + B(x-2)$

$x=6$: $13 = A\cdot 0 + B\cdot 4$ $B = \frac{13}{4}$

$x=2$; $5 = A(-4) + B0$ $A = -\frac{5}{4}$

3. u-sub + half-angle

$$\frac{1}{4}\int 4x^3\sin^2(x^4)\,dx = \frac{1}{4}\int \sin^2(u)\,du = \frac{1}{4}\int\frac{1}{2}\left(1-\cos(2u)\right)du$$

$u = x^4$

$du = 4x^3dx$

$$= \frac{1}{8}u - \frac{1}{8}\frac{1}{2}\int\cos(2u)\,2du = \frac{1}{8}u - \frac{1}{16}\int\cos(w)\,dw$$

$w = 2u$

$dw = 2du$

$$= \frac{1}{8}u - \frac{1}{16}\sin(2u) + C$$

$$= \boxed{\frac{1}{8}x^4 - \frac{1}{16}\sin(2x^4) + C}$$

4. Flow Chart! tan odd, sec odd $\Rightarrow$ pull 1 each out

$$\int \tan^3\theta \sec^3\theta \, d\theta = \int \tan^2\theta \sec^2\theta \underbrace{\sec\theta \tan\theta \, d\theta}_{du}$$

pythag. ~ $\sec^2\theta$

$$= \int (\sec^2\theta - 1)\sec^2\theta \sec\theta \tan\theta \, d\theta$$

$u = \sec\theta$, $du = \sec\theta \tan\theta \, d\theta$

power

$$= \int (u^2-1)u^2 \, du = \int u^4 - u^2 \, du = \frac{u^5}{5} - \frac{u^3}{3} + C$$

$$= \frac{\sec^5\theta}{5} - \frac{\sec^3\theta}{3} + C$$

5. see x^2+4 $\Rightarrow$ $x = 2\tan\theta$

$$\int x^3\sqrt{x^2+4}\,dx =$$

$x^3 = 8\tan^3\theta$

$\sqrt{x^2+4} = \sqrt{4\tan^2\theta + 4} = \sqrt{4(\tan^2\theta+1)} = 2\sqrt{\sec^2\theta} = 2\sec\theta$

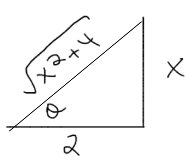
$dx = 2\sec^2\theta \, d\theta$

$$= \int 8\tan^3\theta \cdot 2\sec\theta \cdot 2\sec^2\theta \, d\theta$$

$$= 32\int \tan^3\theta \sec^3\theta \, d\theta = 32\left[\frac{\sec^5\theta}{5} - \frac{\sec^3\theta}{3} + C\right]$$

get x back

$\tan\theta = \frac{x}{2}$ match $\leftrightarrow$



$$32\left[\frac{1}{5}\left(\frac{\sqrt{x^2+4}}{2}\right)^5 - \frac{1}{3}\left(\frac{\sqrt{x^2+4}}{2}\right)^3 + C\right]$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{adj}{hyp}} = \frac{hyp}{adj}$$

6. (you need to do at least three of the following; additional ones may be done for extra credit.)
Evaluate at least three of the following integrals:

(a)

$$\int 2x \tan^{-1} x \, dx =$$

(c) see $\sqrt{x^2-9} \Rightarrow x^2$ can't be < 9

$$\int \frac{\sqrt{x^2-9}}{x^4} \, dx =$$

$x = 3\sec\theta$

$x^2 = 9\sec^2\theta$

$\sqrt{x^2-9} = \sqrt{9(1-\sec^2\theta)} = 3\tan\theta$

$x^4 = 81\sec^4\theta$

$dx = 3\sec\theta\tan\theta$

$= \int \frac{3\tan\theta}{81\sec^4\theta} 3\sec\theta\tan\theta \, d\theta$

$= \frac{1}{27}\int \frac{\tan^2\theta}{\sec^3\theta} \, d\theta$

$= \frac{1}{27}\int \frac{\sin^2\theta}{\cos^2\theta} \cdot \cos^3\theta \, d\theta = \frac{1}{27}\int \sin^2\theta\cos\theta \, d\theta$

$u = \sin\theta$ $\quad \frac{1}{27}\int u^2 \, du$

$du = \cos\theta$

$= \frac{\sin^3\theta}{81} + c$

(d)

$$\int e^{3x} \sin 4x \, dx =$$

(e)

$$\int \frac{x^2}{\sqrt{16-x^2}} \, dx =$$

(f)

$$\int \frac{x^2+4x+6}{x(x^2+2x+1)} \, dx =$$

(d) B/C product $\Rightarrow$ I.B.P. & B/C periodic function (sin) $\Rightarrow$ IBP twice & collect

$$\int e^{3x}\overset{=B}{\sin 4x}\,dx =$$

$u = e^{3x}$ $\quad dv = \sin 4x\, dx$

$du = 3e^{3x}$ $\quad v = -\frac{1}{4}\cos(4x)$

$$= \overset{=A}{-\frac{e^{3x}}{4}\cos(4x)} + \frac{3}{4}\int e^{3x}\cos(4x)\,dx = A + \frac{e^{3x}}{4}\sin 4x - \frac{3}{4}\int \overset{=B}{e^{3x}\sin 4x}$$

$u = e^{3x}$ $\quad dv = \cos 4x$

$du = 3e^{3x}$ $\quad v = \frac{1}{4}\sin 4x$

$$\frac{7}{4}B = A + \frac{e^{3x}}{4}\sin 4x$$

$$\boxed{B = \frac{4}{7}\left(-\frac{e^{3x}}{4}\cos(4x) + \frac{e^{3x}}{4}\sin 4x\right) + C}$$

(a) see product $\Rightarrow$ I.B.P. (LIPET)

$$\int 2x \tan^{-1} x \, dx = \qquad u = \tan^{-1} x \qquad dv = 2x$$

$$du = \frac{1}{1+x^2} \qquad v = x^2$$

$$x^2 \tan^{-1} x - \int \frac{x^2}{1+x^2} dx$$

see $1+x^2$, no u-sub $\Rightarrow$ $x = \tan\theta$

$x^2 = \tan^2\theta$

$1 + x^2 = 1 + \tan^2\theta = \sec^2\theta$

$dx = \sec^2\theta \, d\theta$

$$- \int \frac{\tan^2\theta}{\cancel{\sec^2\theta}} \cancel{\sec^2\theta} \, d\theta$$

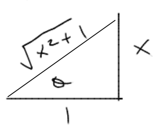
$$x^2 \tan^{-1} x - \int \tan^2\theta \, d\theta$$

$$x^2 \tan^{-1} x - \int \sec^2\theta - 1 \, d\theta = x^2 \tan^{-1} x - \int \sec^2\theta \, d\theta + \int 1 \, d\theta$$

$$= x^2 \tan^{-1} x - \tan\theta + \theta = \boxed{x^2 \tan^{-1} x - x + \tan^{-1}(x) + C}$$

get x back

$x = \tan\theta$



$\theta = \tan^{-1}(x)$