some basics

A sequence $\{a_n\}_0^{+\infty} = a_0, a_1, a_2, \ldots$ **converges** to L if

$$\lim_{n \to +\infty} a_n = L$$

More formally, a sequence $\{a_n\}_0^{+\infty}$ converges to L if, given an $\epsilon > 0$, there exists an N such that for all $n > N$, $|a_n - L| < \epsilon$.

A sequence $\{a_n\}_0^{+\infty} = a_0, a_1, a_2, \ldots$ is **increasing** if
$a_n \leq a_{n+1}$ for all $n \geq 0$.

A sequence $\{a_n\}_0^{+\infty} = a_0, a_1, a_2, \ldots$ is **strictly increasing** if
$a_n < a_{n+1}$ for all $n \geq 0$.

A sequence $\{a_n\}_0^{+\infty} = a_0, a_1, a_2, \ldots$ is **decreasing** if
$a_n \geq a_{n+1}$ for all $n \geq 0$.

A sequence $\{a_n\}_0^{+\infty} = a_0, a_1, a_2, \ldots$ is **strictly decreasing** if
$a_n > a_{n+1}$ for all $n \geq 0$.

A sequence $\{a_n\}_0^{+\infty} = a_0, a_1, a_2, \ldots$ is **monotonic** if
it is either increasing or decreasing.

A **series** is writeen this way:

$$\sum_{n=0}^{+\infty} a_n, \text{ which means } a_0 + a_1 + a_2 + \cdots$$

$a_0, a_1, a_2, \ldots$ is called the **sequence of underlying terms**.

There are two sequences associated with every series:
The sequence $\{s_n\}_0^{+\infty} = s_0, s_1, s_2, \ldots,$ where

$$\begin{aligned} s_0 &= a_0 \\ s_1 &= a_0 + a_1 \\ s_2 &= a_0 + a_1 + a_2 \\ &\vdots \\ s_n &= a_0 + a_1 + a_2 + \cdots + a_n \\ s_{n+1} &= a_0 + a_1 + a_2 + \cdots + a_n + a_{n+1} \end{aligned}$$

is the **sequence of partial sums**.

A series **converges** if and only if its sequence of partial sums converges.