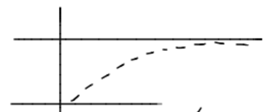
Sanity Check - Sequences & Series

1. What is a sequence?
   a. ordered list of numbers
2. What is an infinite series?
   a. infinite sum : $\sum_{n=1}^{\infty} a_n$
3. Give an example of a convergent sequence.
   a. 1, 1, 1, 1, 1, 1, 1, 1, 1, 1
   b. Give an example of a strictly decreasing convergent sequence.
      i. $\{\frac{1}{x}\}$, $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \longrightarrow 0$
   c. Give an example of a strictly increasing convergent sequence.
      i. $1 - \frac{1}{x} = \frac{x}{x} - \frac{1}{x} = \frac{x-1}{x}$ $\quad \{1 - \frac{1}{x}\}$



4. State the Monotone Convergence Thereom.
   a. Sequence + Bounded + Monotone $\Rightarrow$ Converge

(3 b, c above are such examples)

Monotone

$a_n \leq a_{n+1} \quad \forall\, n$

or

$a_n \geq a_{n+1}$

5. Give an example of a divergent sequence.
   a. $\{n+1\}_{n=1}^{\infty} = 2, 3, 4, 5, 6, \ldots$ or $\{(-1)^n\}$
   b. Give an example of a sequence that diverges to infinity.
      i.
6. Give an example of a convergent series.
   a. $\sum 3(\frac{1}{2})^n \longrightarrow$ geometric $-1 < r < 1$ $\Rightarrow \frac{a}{1-r} \rightarrow \frac{3}{1-\frac{1}{2}} = 6$

   $a_n = 0$

   $\sum_{n=1}^{\infty} a_n = 0 + 0 + 0 \ldots = 0$
7. Give an example of a divergent series.
   a. $\sum_{n=1}^{\infty} (n+1) = 2 + 3 + 4 + 5 + \ldots$
8. What is a geometric series?
   a. ratio of consecutive terms is constant
   b. When does it converge? Why? Formula proved in class
      i.
9. What is the harmonic series?
   a. $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \ldots$
   b. When does it converge? Why?
      i. Never — It's a slowly divergent series (Bernoulli's Proof)
10. How can you tell if a series diverges?
    a. $\sum_{n=1}^{\infty} a_n$ will diverge if $\lim_{n \to \infty} a_n \neq 0$

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$$\lim_{n\to\infty} \frac{4}{2^n} + 16\arctan(n^7)$$

$$=$$

$$\lim_{n\to\infty} \frac{4}{2^n} + 16\underbrace{\lim_{n\to\infty}\arctan(n^7)}$$

$$\downarrow \qquad 16\cdot\arctan\left(\lim_{n\to\infty} n^7\right)$$

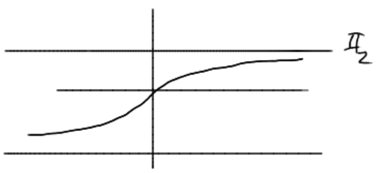
$$0$$

$$=$$

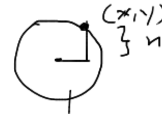
$$16\cdot\underbrace{\arctan}_{\frac{\pi}{2}}(\infty)$$

large + #

what angle gives wicked huge slope



$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x} = \text{slope}$$



rise = y

run = x

	In	Out
tan:	angle	slope
arctan:	slope	angle

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For the series which conv...

(a) $\sum_{n=1}^{\infty} \frac{9^n}{8^n} =$ ______

$\left(\frac{9}{8}\right)^n = \sum_{n=1}^{\infty} ar^n$

(b) $\sum_{n=2}^{\infty} \frac{1}{2^n} =$ ______

$\left(\frac{1}{2}\right)^n$

(c) $\sum_{n=0}^{\infty} \frac{2^n}{6^{2n+1}} =$

$\frac{2^n}{6^{2n+1}} = \frac{2^n}{6^{2n} \cdot 6^1} = \frac{1}{6} \cdot \frac{2^n}{6^{2n}} = \frac{1}{6} \frac{2^n}{(6^2)^n} = \frac{1}{6}\left[\frac{1}{18}\right]^n$

$\frac{1}{6}\left[\frac{2^n}{36^n}\right] = \frac{1}{6}\left(\frac{2}{36}\right)^n$

(d) $\sum_{n=5}^{\infty} \frac{8^n}{9^n} =$

(e) $\sum_{n=1}^{\infty} \frac{6^n}{6^{n+4}} =$

(f) $\sum_{n=1}^{\infty} \frac{8^n + 2^n}{9^n} =$

$\sum \frac{8^n}{9^n} + \sum \frac{2^n}{9^n}$

$\frac{A+B}{C} = \frac{A}{C} + \frac{B}{C}$

$A^N \cdot A^M = A^{N+M}$

$(A^B)^C = A^{BC}$

$\left(\frac{A}{B}\right)^n = \frac{A^n}{B^n}$

$\sum_{\substack{n=0 \\ +1}}^{\infty} \frac{1}{6}\left(\frac{1}{18}\right)^{n-1} = \sum_{n=1}^{\infty} \frac{1}{6}\left(\frac{1}{18}\right)^{n-1}$

$n=0$ $n=1$

$\frac{1}{6}$ $\frac{1}{6 \cdot 18}$