MA 163 — Fri

Series: $\sum_{n=0}^{\infty} a_n = \lim_{N \to \infty} \underbrace{\sum_{n=0}^{N} a_n}_{S_N}$ — partial sums

Geometric Series: $\sum_{n=0}^{\infty} c r^n$, $c = 1^{st}$ term, $r =$ common ratio
$$\begin{cases} \text{DNE} & |r| \geq 1 \\ \frac{c}{1-r} & |r| < 1 \end{cases}$$

Divergence Test: If $\lim_{n \to 0} a_n \neq 0$ then $\sum a_n$ diverges.

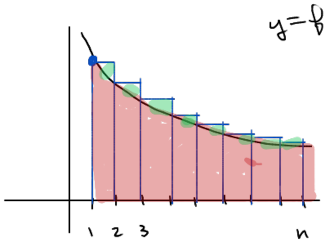
contrapositive

$\boxed{\text{If series converges then } \lim_{n \to \infty} a_n = 0}$

today: Integral Test, Comparison tests (work only for positive (non-negative) series) $a_n \geq 0$

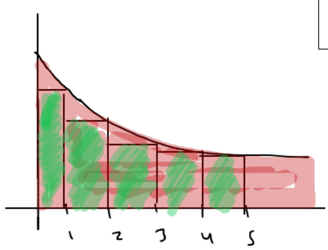
Integral Test

For $\sum_{n=1}^{\infty} a_n$ w/ $a_n = f(n)$ where $f(x)$ is continuous, positive & decreasing:



Relationship b/w area under curve $\int_1^\infty f(x)\,dx$ and the series $\sum_{n=1}^{\infty} a_n$

If $\underbrace{\int_1^\infty f(x)\,dx}_{\text{just the red}} = \infty$ (diverges) then $\underbrace{\sum_{n=1}^{\infty} a_n}_{\text{the red + green}}$ diverges



If $\int_0^\infty f(x)\,dx$ converges then $\sum_{n=0}^{\infty} a_n$ convergers

Ex (10.3.1)

$$\int_{10}^{\infty} \frac{1}{\sqrt{x-9}}\,dx$$

$u = x-9$
$du = dx$
when $x=10$
$u = 10-9 = 1$

$$= \int_1^\infty \frac{du}{\sqrt{u}} = \int_1^\infty u^{-1/2}\,du = 2u^{1/2}\Big|_1^\infty = 2\sqrt{\infty} - 2\sqrt{1} = \infty$$

diverges

Relates to $\sum_{n=10}^{\infty} \frac{1}{\sqrt{n-9}}$

Because $10 > 0$ our rectangles are above curve thus series is "bigger" than the integral (area under curve)

we can apply integral test $\Rightarrow \sum_{n=10}^{\infty} \frac{1}{\sqrt{n-9}}$ diverges