warm-up:

$$\int_1^{10} \frac{3}{x^2}\,dx = 3\int_1^{10} x^{-2}\,dx = \frac{3x^{-1}}{-1} = \frac{-3}{x}\Bigg|_1^{10} = \frac{-3}{10} + 3 = 2\tfrac{7}{10}$$

Repeat for

$$\int_1^{100} \longrightarrow \frac{-3}{x}\Bigg|_1^{100} = \frac{-3}{100} + 3 = 2\tfrac{97}{100}$$

Repeat for

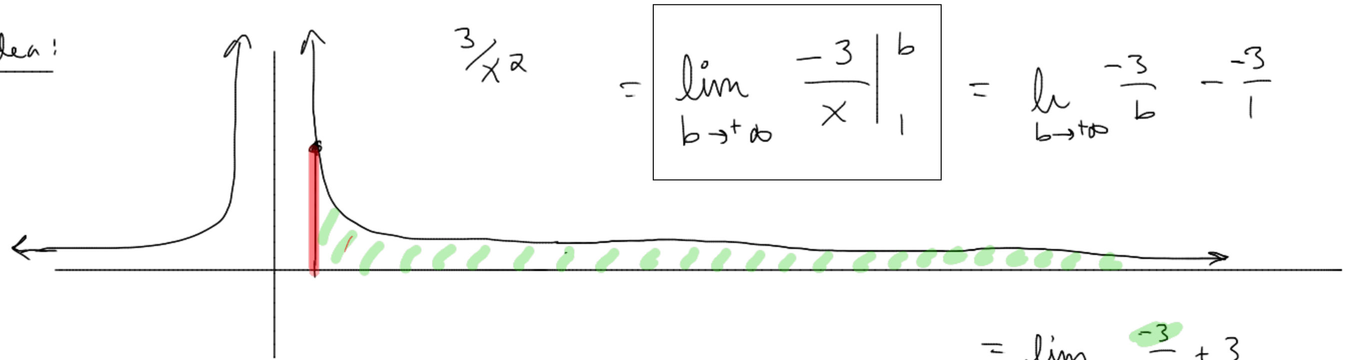
$$\int_1^{1000} \longrightarrow \approx 2.99$$

These are all "proper integrals" (No ∞, No dis<u>continuity</u>)

Improper Integrals

$$\int_1^{+\infty} \frac{3}{x^2}\,dx = \text{How much area is to the right of the line} = \lim_{b \to +\infty} \int_1^b \frac{3}{x^2}\,dx$$

Idea:

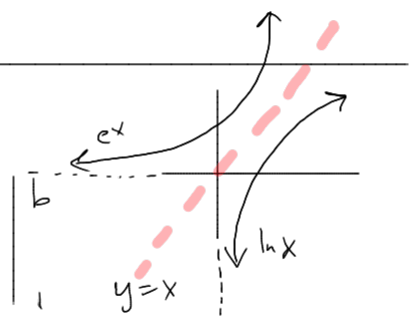


$$= \boxed{\lim_{b\to+\infty} \left.\frac{-3}{x}\right|_1^b} = \lim_{b\to+\infty} \frac{-3}{b} - \frac{-3}{1}$$

$$= \lim_{b\to+\infty} \frac{-3}{b} + 3$$

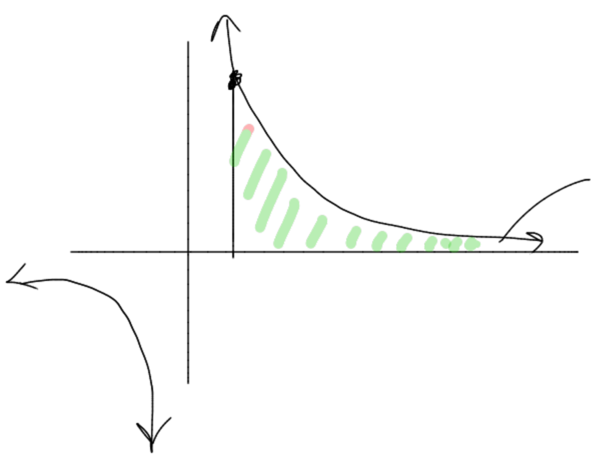
$$= 0 + 3 = \underline{\underline{3}}$$

Ex 2



$$\int_1^{+\infty} \frac{1}{x}\,dx = \lim_{b\to+\infty} \int_1^b \frac{1}{x}\,dx = \lim_{b\to+\infty} \ln|x| \Big|_1^b$$

$$= \lim_{b\to+\infty} \left(\ln(b) - \underbrace{\ln(1)}_{=0}\right) = +\infty - 0 = +\infty$$



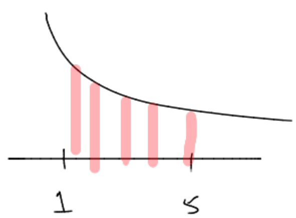
infinite amount of area as $x \to +\infty$

2ND Type

$$\int_1^5 \frac{1}{\sqrt{x-1}}\,dx = \int (x-1)^{-1/2}\,dx = 2(x-1)^{\frac{1}{2}}\Big|$$

@ $x=1$ the integrand is not defined — but it's defined at every other value

$$= \lim_{b\to 1^+}\int_b^5 \frac{1}{\sqrt{x-1}}\,dx = \lim_{b\to 1^+} 2(x-1)^{\frac{1}{2}}\Big|_b^5 = \lim_{x\to 1^+} 2(5-1)^{\frac{1}{2}} - 2(b-1)^{\frac{1}{2}}$$



$$= 4 - 2\underbrace{\lim_{b\to 1^+}(b-1)^{1/2}}_{=0}$$

passed limit across continuous function

$$= 4 - 2\underbrace{\left(\lim_{b\to 1^+}(b) - 1\right)^{1/2}}_{=0} = 4$$

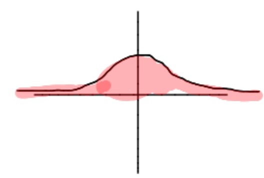
3rd Type:

$$\int_{-\infty}^{\infty} \frac{1}{x^2+1}\,dx = \lim_{b\to+\infty} \int_{-b}^{0} \frac{1}{x^2+1}\,dx + \int_{0}^{b} \frac{1}{x^2+1}\,dx$$

$$= \lim_{b\to+\infty} \tan^{-1}\Big|_{-b}^{0} + \tan^{-1}\Big|_{0}^{b}$$

$\tan^{-1} \leftrightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$

$$= \tan^{-1}(0) - \tan^{-1}(-b) + \tan^{-1}(b) - \tan^{-1}(0)$$



slope

$\tan^{-1}(-\infty)$ $\tan^{-1}(\frac{\pi}{2})$

what angle gives a slope of 0.

angle

$$0 - \left(-\frac{\pi}{2}\right) + \left(\frac{\pi}{2}\right) - 0 = \pi$$

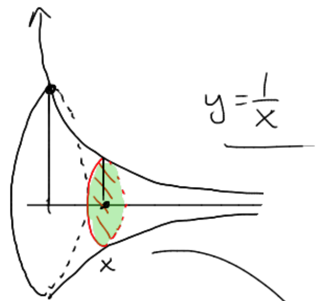
$\tan = \frac{\sin}{\cos} = \frac{y}{x} = \frac{\text{rise}}{\text{run}} = \text{slope}$

Ex. (Volumes)

Consider:

Revolve region under graph b/w $[1,\infty)$ about x-axis

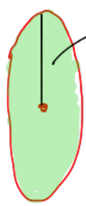
Is the volume of the resulting solid infinite or finite



$y = \frac{1}{x}$

Gabriel's Horn

integrate the area <u>through</u> the solid.



radius $= \frac{1}{x}$

$$\text{Area} = \pi r^2 = \frac{\pi}{x^2}$$

$$V = \int_1^{+\infty} \frac{\pi}{x^2} = \pi \underbrace{\boxed{\int_1^{+\infty} \frac{1}{x^2}\,dx}}_{=1} = \pi$$