MA16? Wk 5 — Mon

Chapter 10: Sequences & Series

Sequences: Ordered List of Numbers

Examples: $\{1,2,3\}$ finite

$\{1,2,1,3,5,1,2,1,3,5,1,2,1,3,5,\dots\}$

$\{a_n\}$ w/ $a_n = \frac{2}{n+1}$ $n=0$, $\Rightarrow \{2, 1, \frac{2}{3}, \frac{2}{4}, \frac{2}{5}, \frac{2}{6}, \dots\}$
n — index; $n=0$

$\{-1,1,-1,1,-1,1,\dots\}$

Most Important Question:
Does the sequence converge?

Def Convergence

A sequence $\{a_n\}$ converges to L (some #) if when you go out far enough in the sequence, you become arbitrarily close to the # L.

— gist —

$\{a_n\} \longrightarrow L$ if $\forall$ (for any) $\epsilon > 0$ (small #), arb.

$|a_n - L| < \epsilon$ whenever $\underbrace{n > M}_{\text{far out in seq.}}$

— formal —

Ex. $\{1,1,2,3,5,8,13,21,34,55,89,144,233,377,\dots\}$
Fibonacci:

Recursively defined: $a_0 = 1$
$a_n = a_{n-1} + a_{n-2}$

Ratios of adjacent terms
$\{\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \frac{55}{34}, \dots$

converges to $\phi = \frac{1+\sqrt{5}}{2}$ golden ratio

One way to determine convergence / divergence of a sequence:

- Look at the continuous function that "relates" to your sequence

your sequence is a subset of the output of a function

Ex $\left\{\frac{1}{n}\right\}_{n=1}^{\infty} = \left\{\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \dots\right\}$

guess: this converges to 0

— change n to x, let x vary over $\mathbb{R}$ —

$f(x) = \frac{1}{x}$

use Calculus (limits & L'Hopital's) $\lim_{x \to \infty} \frac{1}{x} = 0$

so since the sequence is contained inside $\frac{1}{x}$, & $\frac{1}{x} \to 0$ we know $\frac{1}{n} \to 0$.

Ex $\{a_n\}$ $a_n = \sqrt{n} - \sqrt{n+1}$

Does this converge?

$f(x) = \sqrt{x} - \sqrt{x+1}$

$$\lim_{x \to \infty} \sqrt{x} - \sqrt{x+1} = \lim_{x \to \infty} \sqrt{x} - \sqrt{x+1} \cdot \frac{\sqrt{x} + \sqrt{x+1}}{\sqrt{x} + \sqrt{x+1}} = \lim_{x \to \infty} \frac{\overbrace{x-(x+1)}^{-1}}{\sqrt{x} + \sqrt{x+1}}$$

$$= \lim_{x \to \infty} \frac{-1}{\sqrt{x} + \sqrt{x+1}} = \frac{-1}{\infty} = 0$$

Ex $\{(-1)^n\} = \{1, -1, 1, -1, 1, -1, 1, -1, \dots\}$ diverges!

$\quad\;\; = \{a_n\}$

If $\{a_n\} \to L$ then $\{a_n\}$ would be within, say $.25$ of L for n deep in the seq.

Series

Sum of (often infinite) set of terms

use summation notation!

Ex $\displaystyle\sum_{i=1}^{\infty} \frac{1}{i} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$

end index

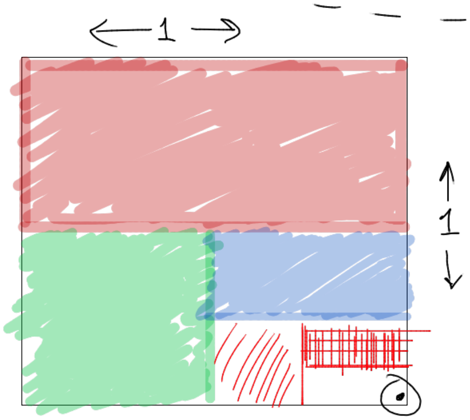
starting index

index

Ex Finite Series: $1 + 2 + 3 + 4 + 5 = \displaystyle\sum_{n=1}^{5} n$

Main Question: Does the series converge or diverge? (this one happens to diverge - more later)

Ex $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots + \frac{1}{2^n} = 1$



Area of Square = 1