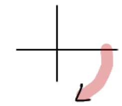
MA 162 wk 5 - Mon: 7.7 & Ch. 10

warm-up:

① Review of Improper Integrals 7.7

② Recall: Integration = Accumulation (Repeated Addition)



angle near very steep neg. slope

$$\boxed{\int_{-\infty}^{\infty}\frac{1}{1+x^2}\,dx} = \tan^{-1}x\Big|_{-\infty}^{\infty} = \lim_{R\to\infty}\tan^{-1}x\Big|_{-R}^{R} = \lim_{R\to\infty}\tan^{-1}(R) - \underbrace{\tan^{-1}(-R)}_{=-\pi/2} = \boxed{\pi}$$

$\tan^{-1}(R)$ " $\tan^{-1}(\infty)$

what angle are you approaching as the slope gets larger.

" $\frac{\pi}{2}$

this integrals accumulates the total change to the angle made as we pass from vertical to vertical

Next Topic: Chapter 10

Ch 10 Sequences & Series :

Sequence: an ordered list of numbers ! Main Question: Converge or Diverge

e.g., 1, 2, 3 (finite)

- $\{a_n\}$ w/ $a_n = n$ w/ $n \in \{1,2,3\}$
- $\{b_n\}$ $b_n = n^2$ w/ $n \in \mathbb{N}$ natural numbers 1, 2, 3, 4, ...

  $= \{1, 4, 9, 16, \ldots\}$
- $\{c_n\}$ $c_n = \frac{1}{n} - \frac{1}{n+1}$ n is the index of the sequence

  general term

- 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ... define this sequence 'recursively'

  Fibonacci Sequence $a_n = a_{n-1} + a_{n-2}$, $a_0 = 1$

  n^{th} term

  Ratios of adjacent terms

  $\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \frac{55}{34}, \frac{89}{55}$ $\rightsquigarrow \phi = \frac{\sqrt{5}+1}{2}$

$a_1 = 1 + 0$

$a_1 = 1$

$a_2 = a_1 + a_0$

$= 2$

How to tell if a sequences converges?

Note: what does it actually mean to "converge"

a sequence {a_n} converges to an number L, if you can go out far enough in the sequence and get arbitrarily close to L

gist

we say $\{a_n\} \longrightarrow L$ if for any number $\epsilon > 0$ (window size) $|a_n - L| < \epsilon$ if $n > M$ for some large number M, (depth level)

math def.

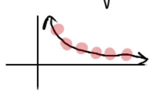
How to tell convergence / divergence?

— look to the function (continuous) that describes your sequence, & use Calc. I Tools (limits & L'Hopital's Rule)

Ex. $\{a_n\}$, $a_n = \frac{1}{n}$ $\{a_n\} = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots\}$

Does $\{a_n\}$ converge? yes $\frac{1}{n}$ gets arbitrarily close to 0 as gets large

The function $f(x) = \frac{1}{x}$ matches our sequence, Since $\lim_{x \to \infty} \frac{1}{x} = 0$, the subset $\{\frac{1}{n}\}_{n \in \mathbb{N}} \to 0$.



Ex. $\{a_n\} = \sqrt{n} - \sqrt{n+1}$

$= \{-1, 1 - \sqrt{2}, \sqrt{2} - \sqrt{3}, \sqrt{3} - \sqrt{4}, \sqrt{4} - \sqrt{5}, \dots\}$

$n=0 \quad n=1 \quad n=2$

$$\lim_{x \to \infty} \sqrt{x} - \sqrt{x+1} = \lim_{x \to \infty} \sqrt{x} - \sqrt{x+1} \cdot \frac{\sqrt{x} + \sqrt{x+1}}{\sqrt{x} + \sqrt{x+1}} = \lim_{x \to \infty} \frac{x - (x+1)}{\sqrt{x} + \sqrt{x+1}}$$

$$= \lim_{x \to \infty} \frac{-1}{\sqrt{x} + \sqrt{x+1}} = \frac{-1}{\underbrace{\sqrt{\infty} + \sqrt{\infty + 1}}_{\infty + \infty = \infty}} = \frac{1}{\infty} = 0$$

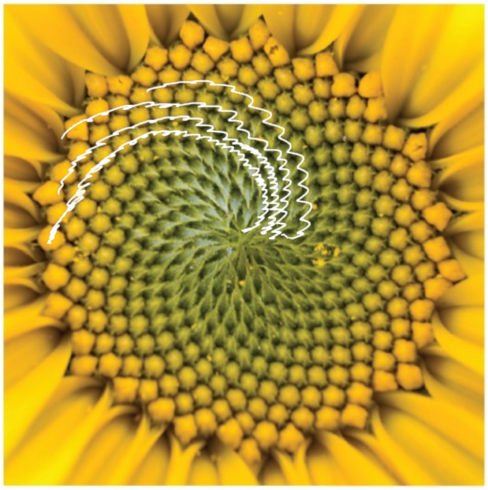
Ex $\{a_n\}$ $\quad a_n = (-1)^n$

$\{1, -1, 1, -1, 1, -1, 1, \ldots\} \longrightarrow$ DNE $\quad$ diverges!

$-1 \quad +1$

Ex $\left\{\dfrac{(-1)^n}{n}\right\}$

$\left\{1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \frac{1}{7}, -\frac{1}{8}, \ldots\right\} \longrightarrow 0$



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