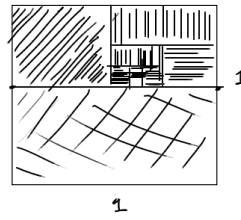MA103   thur   (10.1, 10.2)

Geometric Series.

$$\sum_{n=0}^{\infty} c \cdot r^n$$

c = constant (1st term)

r = common ratio : $\dfrac{n+1 \text{ term}}{n \text{ term}}$: $\dfrac{c \cdot r^{n+1}}{c r^n} = r$

EX

Total Area = 1

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots = 1$

$\parallel$

$c = \frac{1}{2}$, $r = \dfrac{\frac{1}{16}}{\frac{1}{8}} = \frac{8}{16} = \frac{1}{2}$

$= \sum_{n=0}^{\infty} (\frac{1}{2})(\frac{1}{2})^n = \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1}$

Some Geom Series converge, some don't

EX $2 + 4 + 8 + 16 + \cdots = \sum_{n=1}^{\infty} 2^n = \sum_{n=0}^{\infty} 2 \cdot 2^n$

$r = 2$
$\Rightarrow$ diverge.

$\parallel$
$2 + 2^2 + \cdots$

$\parallel \sum_{n=0}^{\infty} 2^n = 2(1 + 2 + 2^2 + \cdots)$

When does a Geom Series converge?

$$\sum_{n=0}^{\infty} c r^n = \begin{cases} \text{converge}^* & |r| < 1 \\ \text{diverge} & |r| \geq 1 \end{cases}$$

* there is a nice
formula for
the limit ...

$\sum_{n=0}^{\infty} 5 \cdot 1^n = \sum_{n=0}^{\infty} 5 = \infty$

$= 5 + 5 + 5 + 5 + \cdots$

to find formula set

$$S = \sum_{n=0}^{\infty} c r^n = c + cr + cr^2 + cr^3 + cr^4 + \cdots$$

so
multi by
r → rS $=$ $cr + cr^2 + cr^3 + cr^4 + \cdots$

subtract! $S - rS$ $= c$ thus factor S, ÷ $S(1-r) = c$

$$\boxed{S = \frac{c}{1-r}}$$ — 1st term
— 1- ratio.

⊛ Caution!   this formula only works if the series converges
... if $|r| < 1$

For example :
$\sum_{n=0}^{\infty} 2^{n+1} = 2 + 4 + 8 + \cdots$; $\begin{array}{l} c = 2 \\ r = 2 \end{array} \Rightarrow S = \dfrac{2}{1-2} = -2$

EX 1   Determine if converges/diverges, if possible find the sum

$\sum_{n=0}^{\infty} 5(\frac{3}{4})^n$ geometric series $|\sum c r^n|$ $\begin{array}{l} c = 5 \\ r = 3/4 \end{array}$ $S = \dfrac{5}{1 - 3/4} = \dfrac{5}{1/4} = 20$

EX 2 $\sum_{n=0}^{\infty} \pi^n = 1 + \pi + \pi^2 + \pi^3 + \cdots$ diverge b/c geometric series, $\begin{array}{l} c = 1 \\ r = \pi > 1 \end{array}$

DIVERGENCE TEST

In order for a series to converge, the n_th term has to get *small* as n grows

If $\lim\limits_{n\to\infty} a_n \neq 0$ then $\sum\limits_{n=1}^{\infty} a_n$ diverges

why? (Proof)

Remember Partial Sum: $S_n = a_1 + a_2 + a_3 + \ldots + a_n$

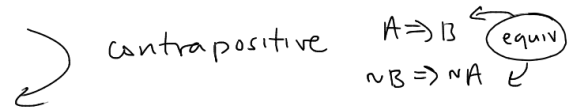subtract ↗ $\underline{\quad S_{n-1} = a_1 + a_2 + a_3 + \ldots + a_{n-1} \quad}$

$S_n - S_{n-1} = a_n$

$$\sum_{n=1}^{\infty} a_n = S \qquad \sum_{n=1}^{\infty} a_n = S$$

If the series converges

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} S_n - S_{n-1} = \lim_{n\to\infty} S_n - \lim_{n\to\infty} S_{n-1} = S - S = 0$$

If converge then $\lim a_n = 0$

$\lim a_n \neq 0 \implies$ diverge

⟩ contrapositive

$A \Rightarrow B \quad$ (equiv)
$\sim B \Rightarrow \sim A$

$\sim A \Rightarrow \sim B$
the converse, not always
equivalent to A => B

$A \Rightarrow B \quad \Big| \quad x < 5$
$\qquad\qquad \Rightarrow$
$\downarrow \qquad\qquad x+1 < 6$
$\sim B \Rightarrow \sim A$

$x + 1 \geqslant 6$
$\Rightarrow x \geqslant 5$

EX $\sum\limits_{n=1}^{\infty} \dfrac{n}{4n+1}$

Think: it diverges, because when n is large the n-th term is approx 1/4, so eventually, it's like you're adding 1/4 over and over and over forever

$\lim\limits_{n\to\infty} \dfrac{n}{4n+1} \overset{L'H}{=} \dfrac{1}{4} \neq 0$

By Div. Test $\Rightarrow$ Diverges

Here the $n^{\underline{th}}$ term does get "small", yet the series still diverges

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}} = \infty$$

EX

—————— p ——————
diverge   converge

0   1