MA163 thur (10.1, 10.2)

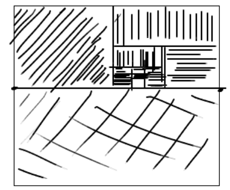
Geometric Series

$\left(\sum_{n=0}^{\infty} c \cdot r^n\right.$

c = constant (1st term)

r = common ratio : $\frac{n+1 \text{ term}}{n \text{ term}} : \frac{c \cdot r^{n+1}}{c r^n} = r$

Ex



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Total Area = 1

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\dots = 1$

$c = \frac{1}{2}$, $r = \frac{\frac{1}{16}}{\frac{1}{8}} = \frac{8}{16} = \frac{1}{2}$

$= \sum_{n=0}^{\infty} (\tfrac{1}{2})(\tfrac{1}{2})^n = \sum_{n=0}^{\infty} (\tfrac{1}{2})^{n+1}$

Some Geom. Series converge, some don't

Ex $2+4+8+16+\dots = \sum_{n=1}^{\infty} 2^n = \sum_{n=0}^{\infty} 2 \cdot 2^n$

$r = 2 \Rightarrow$ diverge. $\quad 2+2^2+\dots \quad 2\sum_{n=0}^{\infty} 2^n = 2(1+2+2^2+\dots)$

When does a Geom Series converge?

$$\sum_{n=0}^{\infty} c r^n = \begin{cases} \text{converge}^{*} & |r|<1 \\ \text{diverge} & |r| \geq 1 \end{cases}$$

* there is a nice formula for the limit ..

$\sum_{n=0}^{\infty} 5 \cdot 1^n = \sum_{n=0}^{\infty} 5 = \infty$

$= 5+5+5+5+\dots$

to find formula set

$S = \sum_{n=0}^{\infty} c r^n = c + cr + cr^2 + cr^3 + cr^4 + \dots$

so mult. by r → $\quad rS = cr + cr^2 + cr^3 \quad cr^4 + \dots$

subtract! $\quad S - rS = c \quad$ thus factor S, ÷ $\quad S(1-r) = c$

$\boxed{S = \frac{c}{1-r}}$ — 1st term / 1 - ratio.

(★) Caution! this formula only works if the series converges

.. if $|r| < 1$

(For example:

$\sum_{n=0}^{\infty} 2^{n+1} = 2+4+8+\dots$; $\quad c = 2, r = 2 \Rightarrow S = \frac{2}{1-2} = -2$

Ex 1 Determine if converges/diverges, if possible find the sum

$\sum_{n=0}^{\infty} 5\left(\frac{3}{4}\right)^n$ geometric series $\left|\sum c r^n\right|$ $\quad c = 5, r = 3/4 \quad S = \frac{5}{1-3/4} = \frac{5}{1/4} = 20$

Ex 2 $\sum_{n=0}^{\infty} \pi^n = 1 + \pi + \pi^2 + \pi^3 + \dots$ diverge b/c geometric series, $c = 1$, $r = \pi > 1$

DIVERGENCE TEST

In order for a series to converge, the n_th term has to get *small* as n grows

If $\lim_{n\to\infty} a_n \neq 0$ then $\sum_{n=1}^{\infty} a_n$ diverges

$A \Rightarrow B$ $x<5 \Rightarrow x+1<6$

$\sim B \Rightarrow \sim A$ $x+1 \geq 6 \Rightarrow x \geq 5$

why: (Proof)

Remember Partial Sum: $S_n = a_1 + a_2 + a_3 + \dots\dots + a_n$

subtract $S_{n-1} = a_1 + a_2 + a_3 + \dots + a_{n-1}$

$S_n - S_{n-1} = a_n$

If the series converges $\sum_{n=1}^{\infty} a_n = S$ $\sum_{n=1}^{\infty} a_n = S$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} S_n - S_{n-1} = \lim_{n\to\infty} S_n - \lim_{n\to\infty} S_{n-1} = S - S = 0$$

If converge then $\lim a_n = 0$

$\lim a_n \neq 0 \Longrightarrow$ diverge

contrapositive $A \Rightarrow B$ equiv $\sim B \Rightarrow \sim A$

$\sim A \Rightarrow \sim B$

the converse, not always equivalent to A => B

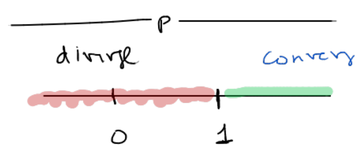
EX $\sum_{n=1}^{\infty} \frac{n}{4n+1}$

$\lim_{n\to\infty} \frac{n}{4n+1} \overset{L'H}{=} \frac{1}{4} \neq 0$

By Div Test $\Rightarrow$ Diverges

Think: it diverges, because when n is large the n-th term is approx 1/4, so eventually, it's like you're adding 1/4 over and over and over forever

Here the n^{th} term does get *small*, yet the series still diverges



EX $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}} = \infty$