10.1 / 10.2 MA163 – Wed

Sequences

: ordered list of #'s

Types:

convergent: $|a_n - L| < \epsilon \quad \forall \quad n \geq M$ (Limit (#); for all; depth)

$\rightarrow$ Ex. $\left\{ \frac{n+1}{n^2} \right\} \longrightarrow 0$

divergent: not

$\rightarrow$ Ex: $\{n+1\} \rightarrow \infty$ (diverges to ∞)

$\rightarrow$ Ex: $\{(-1)^n\} \rightarrow \{-1, 1, -1, 1, \ldots\}$ diverges

bounded: $|a_n| \leq M$ ← some #

increasing: $a_n < a_{n+1}$

decreasing: $a_n > a_{n+1}$

monotone: either increasing or decreasing

Ex $\left\{ \frac{1}{n} \right\}_{n=1}^{\infty}$ is bounded & decreasing & convergent

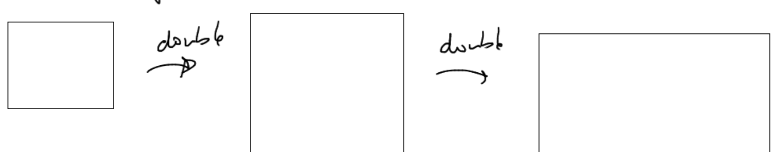
$\left\{ \left(\frac{-1}{2}\right)^n \right\}_{n=0}^{\infty}$ is not monotone, bounded & convergent

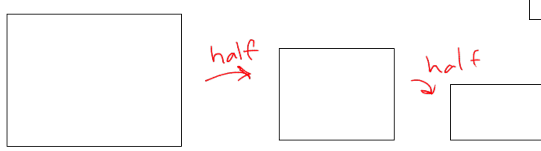
$\{5, 4, 5, 4, 5, 4, \ldots\}$ not monotone; divergent and bounded

Theorem: Bounded Monotone Convergence

If a seq. is bounded & monotone then it converges.

geometric: ratio of consecutive terms is constant





ratio $= \frac{1}{2}$

Ex $\left\{5, 5\left(\frac{1}{2}\right), 5\left(\frac{1}{4}\right), 5\left(\frac{1}{8}\right), 5\left(\frac{1}{16}\right), \ldots 5 \cdot \left(\frac{1}{2^n}\right)\right\}$

Ratio $\dfrac{5 \cdot \left(\frac{1}{2^n}\right)}{5 \cdot \left(\frac{1}{2^{n+1}}\right)} = \dfrac{\frac{1}{2^n}}{\frac{1}{2^{n+1}}} = \dfrac{2^{n+1}}{2^n} = 2$

the n_th term is twice larger than the n+1 termv

Series: Sum of a set of terms

(Main question: Does a given series converge? Yes/No

the answer depends on the convergence / divergence of a related sequence)

How to tell if a series converges

1. Form the sequence of its partial sums

given: $\sum_{n=0}^{\infty} a_n = \lim_{N \to \infty} \sum_{n=0}^{N} a_n$

$S_0 = a_0 \longrightarrow$ # b_0

evaluate sum (series) up to index

1st partial sum

$S_1 = a_0 + a_1 \longrightarrow$ # b_1

$S_2 = a_0 + a_1 + a_2 \longrightarrow$ this sum = b_2

$S_3 = a_0 + a_1 + a_2 + a_3 \longrightarrow$ b_3

$\{b_n\}$

2. If $\{b_n\}$ "the sequence of partial sums" converges then the series converges

likewise if $\{b_n\}$ diverge, series diverges.

Telescoping Series

Ex
$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \ldots$$
$$= \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \ldots$$

write
$$\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1} \Rightarrow 1 = A(n+1) + Bn = \underbrace{(A+B)}_{=0}n + A$$

$A = 1$
$B = -1$

① $$= \sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1}$$

stop here ☆

② Partial Sum : $S_1 : \frac{1}{1} - \frac{1}{1+1} = 1 - \frac{1}{2} = \frac{1}{2}$

plug in 1 and 2 ← $S_2 : \frac{1}{2} + \frac{1}{2} - \frac{1}{3} = 1 - \frac{1}{3}$

$S_1 + a_2$

better

$S_2 : \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) = 1 - \frac{1}{3}$

$S_3 : \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) = 1 - \frac{1}{4}$

$S_4 : \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) = 1 - \frac{1}{5}$

$S_n = 1 - \frac{1}{n+1}$ ← n^{th} term of Partial Sum. Take Limit

$$\boxed{\lim_{n\to\infty} 1 - \frac{1}{n+1} = 1}$$

Ans: series converges to 1

Next

$$\sum_{n=1}^{\infty} \frac{1}{n} = \overbrace{\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{100}}^{\approx 5.18}$$

harmonic series