

Sequence: ordered list of #'s, $\{a_n\}$

Types:

• Convergent: $|a_n - L| < \epsilon$ for all $n > M$
(cont.) limit
for all $n > M$ ← depth in seq.
 ϵ small # arbitrary

$$\left\{ \frac{1}{n} \right\} \rightarrow 0 \text{ as } n \rightarrow \infty$$

• Divergent: not

$$\{n+1\} \rightarrow \infty \text{ diverges to } \infty$$

$$\{(-1)^n\} \text{ diverges}$$

• Bounded: \exists a # M s.t. $|a_n| \leq M$

Ex: $\left\{ \frac{100}{n^2} \right\}_{n=1}^{\infty}$ bounded since $\frac{100}{n^2} \leq 100$

• Strictly Increasing: $a_n < a_{n+1}$

Ex $\{n+1\}$ is inc.

• Decreasing: $a_n > a_{n+1}$

Ex $\left\{ \frac{1}{n} \right\}_{n=1}^{\infty}$ is decreasing

• Monotone: either inc. or dec.

Theorem: Bounded-Monotone Convergence

Any sequence that is bounded & monotone must converge

Ex $\left\{ \frac{1}{n} \right\}$ Bounded, $\frac{1}{n} < 1$ $\forall n \geq 1$
 Monotone $\frac{1}{n} > \frac{1}{n+1}$ } $\lim_{n \rightarrow \infty} \frac{1}{n} = 0$

• Geometric Sequence: ratio of consecutive terms is constant

Ex $5, 5 \cdot \frac{3}{4}, 5 \cdot \left(\frac{3}{4}\right)^2, 5 \cdot \left(\frac{3}{4}\right)^3, \dots \rightarrow \frac{5 \cdot \left(\frac{3}{4}\right)^n}{5 \cdot \left(\frac{3}{4}\right)^{n-1}} = \frac{3}{4} = \text{constant ratio}$

Does this sequence $\left\{ 5 \left(\frac{3}{4}\right)^n \right\}$ converge? $\lim_{n \rightarrow \infty} 5 \left(\frac{3}{4}\right)^n \text{ as } n \rightarrow \infty \left(\frac{3}{4}\right)^n \rightarrow 0$, \therefore **yes**

Ex $\left\{ \frac{1}{2^n} \right\}_{n=0}^{\infty} = \left\{ 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots \right\}$ $\frac{1}{8} = \frac{\frac{1}{2^3}}{\frac{1}{2^2}} = \frac{2^2}{2^3} = \frac{1}{2} = \text{common ratio}$

Series: a sum of terms (numbers) || general Question: Does the series converge

Ex $1 + 2 + 3 + 4 + 5 + \dots = \sum_{n=1}^{\infty} n$
 ∞ ← end
 $n=1$ ↑ start

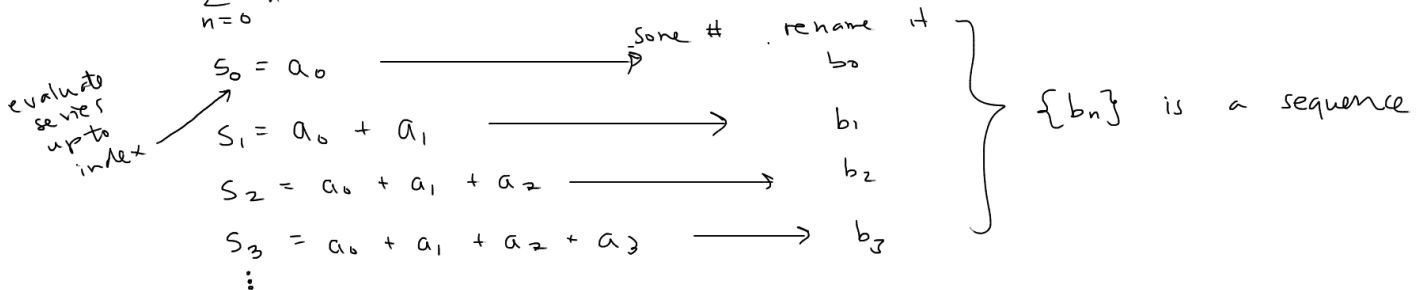
Ex $\sum_{n=1}^{\infty} \frac{1}{n} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \dots$ we'll see ∞
 Harmonic Series

Ex $\sum_{n=1}^{\infty} \frac{1}{n^2}$ we'll see finite = $\frac{\pi^2}{6}$

How tell if a series converges:

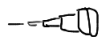
① Sequence of Partial Sums

i.e., $\sum_{n=0}^{\infty} a_n$



② Series converges if its sequence of partial sums converges.
 diverges " " " " " " diverges

Telescoping Series \square



$$\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$$

$$1 = A(n+1) + B(n) = (A+B)n + A$$

$A+B=0, A=1$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right)$$

partial sums

$$S_1 = \text{eval series up to } n=1 = \frac{1}{2}$$

$$S_2 = a_1 + a_2 = \frac{1}{2} + \frac{1}{2(2+1)} = \frac{1}{2} + \frac{1}{6} = \frac{2}{3} = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) = 1 - \frac{1}{3}$$

$$S_3 = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} = \frac{9}{12} = \frac{3}{4}$$

$$S_4 = \dots$$

\vdots

$$S_n = 1 - \frac{1}{n+1}$$

this S_n is the n -th partial sum. if it converges as n grows, our series converges

$$\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n+1} \right) = 1 \Rightarrow \underline{\text{series converges}}$$

since we can't see pattern
alternate expression

$$= 1 - \frac{1}{2}$$

$$= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) = 1 - \frac{1}{4}$$

$$= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) = 1 - \frac{1}{5}$$

use this to find a pattern...

$$\sum_{n=1}^{\infty} a_n = \lim_{n \rightarrow \infty} S_n$$

$S_n = \text{partial sum.}$

geometric series

! to memorize!

$$\sum_{n=0}^{\infty} cr^n =$$