10.1 / 10.2 MA163 Wed

Sequence: ordered list of #'s, $\{a_n\}$

Types:

- Convergent: $|a_n - L| < \epsilon \quad \forall \; n \geq M$ — (cont) limit; ϵ small # arbitrary; for all; M depth in seq.

  $\{\frac{1}{n}\} \to 0$ as $n \to \infty$

- Divergent: not ↑

  $\{n+1\} \to \infty$ diverges to ∞

  $\{(-1)^n\}$ diverges

- Bounded: $\exists$ (there exists) a # M s.t. $|a_n| \leq M$

  Ex: $\{\frac{100}{n^2}\}_{n=1}^{\infty}$ bounded since $\frac{100}{n^2} \leq 100$

- Strictly Increasing: $a_n < a_{n+1}$

  Ex $\{n+1\}$ is inc.

- Decreasing $a_n > a_{n+1}$

  Ex $\{\frac{1}{n}\}_{n=1}^{\infty}$ is decreasing

- Monotone: either inc. or dec.

Theorem: Bounded-Monotone Convergence:
Any sequence that is bounded & monotone must converge

Ex $\{\frac{1}{n}\}$ Bounded, $\frac{1}{n} < 1 \;\; \forall \; n \geq 1$
Monotone $\frac{1}{n} > \frac{1}{n+1}$
$\Big\}$ $\lim_{n\to\infty} \frac{1}{n} = 0$

- Geometric Sequence: ratio of consecutive terms is constant

Ex $5, 5\cdot\frac{3}{4}, 5\cdot(\frac{3}{4})^2, 5\cdot(\frac{3}{4})^3, \ldots \Rightarrow \frac{5\cdot(\frac{3}{4})^n}{5\cdot(\frac{3}{4})^{n-1}} = \frac{3}{4} =$ constant ratio

Does this sequence $\{5(\frac{3}{4})^n\}$: $\lim_{n\to\infty} 5(\frac{3}{4})^n$ as $n\to\infty$ $(\frac{3}{4})^n \to 0$, so yes

Ex $\{\frac{1}{2^n}\}_{n=0}^{\infty} = \{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots\}$ $\quad \frac{\frac{1}{8}}{\frac{1}{4}} = \frac{\frac{1}{2^3}}{\frac{1}{2^2}} = \frac{2^2}{2^3} = \frac{1}{2} =$ common ratio

Series : a sum of terms (numbers) ‖ general Question; Does the series converge

Ex $1+2+3+4+5+\dots = \sum_{n=1}^{\infty} n$ (∞ ← end, $n=1$ ← start)

Ex $\sum_{n=1}^{\infty} \frac{1}{n} = \frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\dots \underset{\text{see}}{\overset{\text{we'll}}{=}} \infty$

Harmonic Series

Ex $\sum_{n=1}^{\infty} \frac{1}{n^2} \underset{\text{see}}{\overset{\text{we'll}}{=}}$ finite $= \frac{\pi^2}{6}$

How tell if a series converges :

1. Sequence of Partial Sums

i.e., $\sum_{n=0}^{\infty} a_n$

evaluate series up to index

		some #	rename it
$S_0 = a_0$	⟶		b_0
$S_1 = a_0 + a_1$	⟶		b_1
$S_2 = a_0 + a_1 + a_2$	⟶		b_2
$S_3 = a_0 + a_1 + a_2 + a_3$	⟶		b_3
⋮			

$\{b_n\}$ is a sequence

2. Series converges if its sequence of partial sums converges.

diverges " " " " " " diverges

Telescoping Series

$$\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$$

$$1 = A(n+1) + B(n) = (A+B)n + A$$

$$A+B=0, \quad A=1$$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1}$$

partial sums

since we can't see pattern
alternate expression

S_1 = eval series up to $n=1$ $= \frac{1}{2}$ $\quad = 1 - \frac{1}{2}$

$S_2 = a_1 + a_2 = \frac{1}{2} + \frac{1}{2(2+1)} = \frac{1}{2} + \frac{1}{6} = \frac{2}{3} = \left(1-\frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) = 1 - \frac{1}{3}$

$S_3 = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} = \frac{9}{12} = \frac{3}{4}$ $\quad = \left(1-\frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) = 1 - \frac{1}{4}$

$S_4 = \ldots$ $\quad = \left(1-\frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) = 1 - \frac{1}{5}$

$\vdots$

use this to find a pattern...

$$S_n = 1 - \frac{1}{n+1}$$

this S_n is the n_th partial sum. if it converges as n grows, our series converges

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} S_n$$

S_n = partial sum.

$$\lim_{n\to\infty} 1 - \frac{1}{n+1} = 1 \Rightarrow \underline{\text{series converges}}$$

geometric series ! tomorrow !

$$\sum_{n=0}^{\infty} cr^n =$$