(EX) 10.1.18 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | DEFAULT FEEDBACK

Determine the limit of the sequence or state that the sequence diverges.

$$a_n = \frac{8 + n - 2n^2}{9n^2 + 2}$$

(Use symbolic notation and fractions where needed. Enter DNE if the sequence diverges.)

For $a_n = f(n), f(x) = \frac{8+x-2x^2}{9x^2+2}$. Thus,

$$\lim_{x\to\infty}\left(\frac{8 + x - 2x^2}{9x^2 + 2}\right) = \lim_{x\to\infty}\left(\frac{\frac{8}{x^2} + \frac{1}{x} - 2}{9 + \frac{2}{x^2}}\right) = -\frac{2}{9}$$

Therefore, the sequence converges.

(EX) 10.1.17 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | DEFAULT FEEDBACK

Determine the limit of the sequence and state if the sequence converges or diverges.

$$b_n = \frac{2n - 4}{4n + 4}$$

(Use symbolic notation and fractions where needed. Enter DNE if the sequence diverges.)

Solution

For $b_n = f(n),\ f(x) = \frac{2x-4}{4x+4}$. Thus,

$$\lim_{x\to\infty}\left(\frac{2x - 4}{4x + 4}\right) = \lim_{x\to\infty}\left(\frac{2 - \frac{4}{x}}{4 + \frac{4}{x}}\right) = \frac{1}{2}$$

Therefore, the sequence converges.

(EX) 10.1.16 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | DEFAULT FEEDBACK

Determine the limit of the sequence or state that the sequence diverges.

$$a_n = 9 - \frac{8}{n^2}$$

(Use symbolic notation and fractions where needed. Enter DNE if the sequence diverges.)

Solution

For $a_n = f(n), f(x) = 9 - \frac{8}{x^2}$. Thus,

$$\lim_{x\to\infty}\left(9 - \frac{8}{x^2}\right) = 9 - 0 = 9$$

Therefore, the sequence converges.

(EX) 10.1.18 (Rogawski 4e ET) (ESF)

QUESTION VIEW | SOLUTION | INCORRECT FEEDBACK - 1 | DEFAULT FEEDBACK

Determine the limit of the sequence or state that the sequence diverges.

$$a_n = \frac{4 + n - 3n^2}{4n^2 + 3}$$

(Use symbolic notation and fractions where needed. Enter DNE if the sequence diverges.)

Solution

For $a_n = f(n), f(x) = \frac{4+x-3x^2}{4x^2+3}$.

$$\lim_{x\to\infty}\left(\frac{4 + x - 3x^2}{4x^2 + 3}\right) = \lim_{x\to\infty}\left(\frac{\frac{4}{x^2} + \frac{1}{x} - 3}{4 + \frac{3}{x^2}}\right) = -\frac{3}{4}$$

Therefore, the sequence converges.

(EX) 10.1.48 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | DEFAULT FEEDBACK

Use the appropriate limit laws and theorems to determine the limit of the sequence.

$$a_n = \frac{9\sqrt{n}}{8\sqrt{n} + 7}$$

(Use symbolic notation and fractions where needed. Enter DNE if the sequence diverges.)

Solution

Divide the numerator and denominator by $\sqrt{n}$.

$$\lim_{n\to\infty}\frac{9\sqrt{n}}{8\sqrt{n} + 7} = \lim_{n\to\infty}\frac{9\frac{\sqrt{n}}{\sqrt{n}}}{8\frac{\sqrt{n}}{\sqrt{n}} + \frac{7}{\sqrt{n}}}$$

$$= \lim_{n\to\infty}\frac{9}{8 + \frac{7}{\sqrt{n}}}$$

$$= \lim_{x\to\infty}\frac{9}{8 + \frac{7}{\sqrt{x}}}$$

$$= \frac{9}{8}$$

(EX) 10.1.14 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | INCORRECT FEEDBACK - 1 | DEFAULT FEEDBACK

Suppose that $\lim_{n\to\infty} a_n = 9$ and $\lim_{n\to\infty} b_n = 5$.

Determine the limits.

$\lim_{n\to\infty}(a_n + b_n) =$ 14

QUESTION VIEW | SOLUTION | INCORRECT FEEDBACK - 1 | DEFAULT FEEDBACK

Apply the Limit Laws of Sequences.

$$\lim_{n\to\infty}(a_n + b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n = 9 + 5 = 14$$

$$\lim_{n\to\infty} a_n^3 = \lim_{n\to\infty}(a_n \cdot a_n \cdot a_n) = \lim_{n\to\infty} a_n \cdot \lim_{n\to\infty} a_n \cdot \lim_{n\to\infty} a_n = \left(\lim_{n\to\infty} a_n\right)^3 = 9^3 = 729$$

$$\lim_{n\to\infty}\cos(\pi b_n) = \cos\left(\lim_{n\to\infty}\pi b_n\right) = \cos\left(\pi\lim_{n\to\infty} b_n\right) = \cos(5\pi) = -1$$

$$\lim_{n\to\infty}(a_n^2 - 2a_nb_n) = \lim_{n\to\infty} a_n^2 - \lim_{n\to\infty} 2a_nb_n = \left(\lim_{n\to\infty} a_n\right)^2 - 2\lim_{n\to\infty} a_n \cdot \lim_{n\to\infty} b_n = 9^2 - 2\cdot 9\cdot 5 = -9$$

$\lim_{n\to\infty}(a_n^2 - 2a_nb_n) =$ −9

(EX) 10.1.54 (Rogawski 4e ET) Ro

QUESTION VIEW SOLUTION DEFAULT FEEDBACK

Use the appropriate limit laws and theorems to determine the limit of the sequence.

$$a_n = \tan^{-1}\left(1 - \frac{9}{n}\right)$$

(Use symbolic notation and fractions where needed. Enter DNE if the sequence diverges.)

Solution

Since $f(x) = \tan^{-1} x$ is a continuous function and $a_n = f\left(1 - \frac{9}{x}\right)$, it follows that

$$\begin{aligned}\lim_{n\to\infty} a_n &= f\left(\lim_{n\to\infty}\left(1 - \frac{9}{n}\right)\right)\\ &= \tan^{-1}\left(\lim_{n\to\infty}\left(1 - \frac{9}{n}\right)\right)\\ &= \tan^{-1}\left(\lim_{x\to\infty}\left(1 - \frac{9}{x}\right)\right)\\ &= \tan^{-1} 1\\ &= \frac{\pi}{4}\end{aligned}$$

(EX) 10.1.53 (Rogawski 4e ET) Rogawsl

QUESTION VIEW SOLUTION DEFAULT FEEDBACK

Use the appropriate limit laws and theorems to determine the limit of the sequence.

$$a_n = \left(3 + \frac{7}{n^2}\right)^{1/4}$$

(Use symbolic notation and fractions where needed. Enter DNE if the sequence diverges.)

Solution

Since $f(x) = x^{1/4}$ is a continuous function and $f\left(3 + \frac{7}{n^2}\right) = a_n$, it follows that

$$\begin{aligned}\lim_{n\to\infty}\left(3 + \frac{7}{n^2}\right)^{1/4} &= f\left(\lim_{n\to\infty}\left(3 + \frac{7}{n^2}\right)\right)\\ &= \left(\lim_{n\to\infty}\left(3 + \frac{7}{n^2}\right)\right)^{1/4}\\ &= \left(\lim_{x\to\infty}\left(3 + \frac{7}{x^2}\right)\right)^{1/4}\\ &= (3+0)^{1/4}\\ &= 3^{1/4}\end{aligned}$$

(EX) 10.1.29 (Rogawski 4e ET) Rogawski

QUESTION VIEW SOLUTION DEFAULT FEEDBACK

Determine the limit of the sequence.

$$a_n = \sqrt{100 + \frac{1}{n}}$$

(Use symbolic notation and fractions where needed. Enter DNE if the sequence diverges.)

Solution

First, integrate the radicand. Note that $100 + \frac{1}{n} = f(n)$, where $f(x) = 100 + \frac{1}{x}$. Therefore,

$$\begin{aligned}\lim_{n\to\infty}\left(100 + \frac{1}{n}\right) &= \lim_{x\to\infty}\left(100 + \frac{1}{x}\right)\\ &= \lim_{x\to\infty} 100 + \lim_{x\to\infty}\frac{1}{x}\\ &= 100\end{aligned}$$

With $g(x) = \sqrt{x}$, $a_n = g\left(100 + \frac{1}{n}\right)$. Since g is a continuous function for $x > 0$,

$$\begin{aligned}\lim_{n\to\infty} a_n &= \lim_{n\to\infty} g\left(100 + \frac{1}{n}\right)\\ &= g\left(\lim_{n\to\infty}\left(100 + \frac{1}{n}\right)\right)\\ &= g(100)\\ &= \sqrt{100}\end{aligned}$$

(EX) 10.1.30 (Rogawski 4e ET) R

QUESTION VIEW SOLUTION DEFAULT FEEDBACK

Determine the limit of the sequence.

$$a_n = e^{7n/(8n+5)}$$

(Use symbolic notation and fractions where needed. Enter DNE if the sequence diverges.)

Solution

$$\begin{aligned}\lim_{n\to\infty}\frac{7n}{8n+5} &= \lim_{x\to\infty}\frac{7x}{8x+5}\\ &= \lim_{x\to\infty}\frac{7}{8 + \frac{5}{x}}\\ &= \frac{7}{8}\end{aligned}$$

With $g(x) = e^x$, $a_n = g\left(\frac{7n}{8n+5}\right)$. Since g is a continuous function,

$$\begin{aligned}\lim_{n\to\infty} a_n &= \lim_{n\to\infty} g\left(\frac{7n}{8n+5}\right)\\ &= g\left(\lim_{n\to\infty}\frac{7n}{8n+5}\right)\\ &= g\left(\frac{7}{8}\right)\end{aligned}$$

(EX) 10.1.55 (Rogawski 4e ET) R

QUESTION VIEW SOLUTION DEFAULT FEEDBACK

Use the appropriate limit laws and theorems to determine the limit of the sequence.

$$a_n = \ln\left(\frac{8n^2 + 9n + 7}{9n^2 + 5n + 8}\right)$$

(Use symbolic notation and fractions where needed. Enter DNE if the sequence diverges.)

Solution

Since $f(x) = \ln x$ is a continuous function, it follows that

$$\begin{aligned}\lim_{n\to\infty} a_n &= \lim_{n\to\infty}\ln\left(\frac{8n^2 + 9n + 7}{9n^2 + 5n + 8}\right)\\ &= \lim_{x\to\infty}\ln\left(\frac{8x^2 + 9x + 7}{9x^2 + 5x + 8}\right)\\ &= \ln\left(\lim_{x\to\infty}\left(\frac{8x^2 + 9x + 7}{9x^2 + 5x + 8}\right)\right)\\ &= \ln\left(\lim_{x\to\infty}\left(\frac{8 + \frac{9}{x} + \frac{7}{x^2}}{9 + \frac{5}{x} + \frac{8}{x^2}}\right)\right)\\ &= \ln\frac{8}{9}\end{aligned}$$

(EX) 10.1.52 (Rogawski 4e ET) R

QUESTION VIEW SOLUTION DEFAULT FEEDBACK

Use the appropriate limit laws and theorems to determine the limit of the sequence.

$$d_n = \ln\left(n^2 + 1\right) - \ln\left(n^2 - 1\right)$$

(Use symbolic notation and fractions where needed. Enter DNE if the sequence diverges.)

Solution

Note that

$$d_n = \ln\left(\frac{n^2 + 1}{n^2 - 1}\right) = \ln\left(\frac{1 + 1/n^2}{1 - 1/n^2}\right)$$

Thus, since $f(x) = \ln(x)$ is a continuous function and $f\left(\frac{1+1/n^2}{1-1/n^2}\right) = d_n$,

$$\begin{aligned}\lim_{n\to\infty} d_n &= f\left(\lim_{n\to\infty}\frac{1 + \frac{1}{n^2}}{1 - \frac{1}{n^2}}\right)\\ &= \ln\left(\frac{\lim_{n\to\infty}\left(1 + \frac{1}{n^2}\right)}{\lim_{n\to\infty}\left(1 - \frac{1}{n^2}\right)}\right)\\ &= \ln\left(\frac{\lim_{x\to\infty}\left(1 + \frac{1}{x^2}\right)}{\lim_{x\to\infty}\left(1 - \frac{1}{x^2}\right)}\right)\\ &= \ln\left(\frac{1}{1}\right)\\ &= 0\end{aligned}$$

(EX) 10.1.39 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | DEFAULT FEEDBACK

Use the appropriate limit laws and theorems to determine the limit of the sequence.

$$a_n = 9 + \left(-\frac{1}{9}\right)^n$$

(Use symbolic notation and fractions where needed. Enter DNE if the sequence diverges.)

Solution

Use the Limit Laws for Sequences.

$$\lim_{n\to\infty}\left(9+\left(-\frac{1}{9}\right)^n\right) = \lim_{n\to\infty} 9 + \lim_{n\to\infty}\left(-\frac{1}{9}\right)^n = 9 + \lim_{n\to\infty}\left(-\frac{1}{9}\right)^n$$

Now,

$$-\left(\frac{1}{9}\right)^n \le \left(-\frac{1}{9}\right)^n \le \left(\frac{1}{9}\right)^n$$

Since $0 \le \frac{1}{9} < 1$,

$$\lim_{n\to\infty}\left(-\left(\frac{1}{9}\right)^n\right) = \lim_{n\to\infty}\left(\frac{1}{9}\right)^n = 0$$

Hence, by the Squeeze Theorem,

$$\lim_{n\to\infty}\left(-\frac{1}{9}\right)^n = 0$$

Thus,

$$\lim_{n\to\infty}\left(9+\left(-\frac{1}{9}\right)^n\right) = 9 + 0 = 9$$

(EX) 10.1.42 (Rogawski 4e ET) (ESF)

QUESTION VIEW | SOLUTION | INCORRECT FEEDBACK - 1 | DEFAULT FEEDBACK

Use the appropriate limit laws and theorems to determine the limit of the sequence.

$$b_n = e^{11-n^2}$$

(Give an exact answer. Use symbolic notation and fractions where needed. Enter DNE if the sequence diverges.)

Solution

Since $b_n = f(n)$, where $f(x) = e^{11-x^2}$, it follows that

$$\lim_{n\to\infty} e^{11-n^2} = \lim_{x\to\infty} e^{11-x^2} = \lim_{x\to\infty}\frac{e^{11}}{e^{x^2}} = 0$$

(EX) 10.1.50 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | DEFAULT FEEDBACK

Use the appropriate limit laws and theorems to determine the limit of the sequence.

$$c_n = \frac{(-1)^n}{\sqrt{5n}}$$

(Use symbolic notation and fractions where needed. Enter DNE if the sequence diverges.)

Solution

Note that

$$-\frac{1}{\sqrt{5n}} \le \frac{(-1)^n}{\sqrt{5n}} \le \frac{1}{\sqrt{5n}}$$

Besides,

$$\lim_{n\to\infty}\left(-\frac{1}{\sqrt{5n}}\right) = \lim_{n\to\infty}\frac{1}{\sqrt{5n}} = 0$$

Apply the Squeeze Theorem for Sequences to conclude that

$$\lim_{n\to\infty}\frac{(-1)^n}{\sqrt{5n}} = 0$$

(EX) 10.1.49 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | DEFAULT FEEDBACK

Use the appropriate limit laws and theorems to determine the limit of the sequence.

$$a_n = \frac{2\cos(n)}{n}$$

(Use symbolic notation and fractions where needed. Enter DNE if the sequence diverges.)

Solution

Since $-2 \le 2\cos(n) \le 2$, it follows that

$$-\frac{2}{n} \le \frac{2\cos(n)}{n} \le \frac{2}{n}$$

Note that

$$\lim_{n\to\infty}\left(-\frac{2}{n}\right) = \lim_{n\to\infty}\frac{2}{n} = 0$$

Apply the Squeeze Theorem for Sequences to conclude that

$$\lim_{n\to\infty}\frac{2\cos(n)}{n} = 0$$

(EX) 10.1.59 (Rogawski 4e ET) (Video Feedback)

QUESTION VIEW | SOLUTION | DEFAULT FEEDBACK

Use the appropriate limit laws and theorems to determine the limit of the sequence.

$$y_n = \frac{e^n + (-2)^n}{11^n}$$

(Use symbolic notation and fractions where needed. Enter DNE if the sequence diverges.)

Solution

Rewrite y_n as

$$\frac{e^n + (-2)^n}{11^n} = \left(\frac{e}{11}\right)^n + \left(-\frac{2}{11}\right)^n$$

Hence,

$$\lim_{n\to\infty}\frac{e^n + (-2)^n}{11^n} = \lim_{n\to\infty}\left(\frac{e}{11}\right)^n + \lim_{n\to\infty}\left(-\frac{2}{11}\right)^n$$

assuming both limits on the right-hand side exist.

Note that

$$-\left(\frac{2}{11}\right)^n \le \left(-\frac{2}{11}\right)^n \le \left(\frac{2}{11}\right)^n$$

By the Limit of Geometric Sequences, since $0 < \frac{2}{11} < 1$ and $0 < \frac{e}{11} < 1$,

$$\lim_{n\to\infty}\left(\frac{e}{11}\right)^n = \lim_{n\to\infty}\left(\frac{2}{11}\right)^n = 0$$

Therefore, by the Squeeze Theorem, $\lim_{n\to\infty}\left(-\frac{2}{11}\right)^n = 0$

Thus,

$$\lim_{n\to\infty} y_n = \lim_{n\to\infty}\left(\frac{e}{11}\right)^n + \lim_{n\to\infty}\left(-\frac{2}{11}\right)^n = 0$$

(EX) 10.1.67 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | DEFAULT FEEDBACK

Find the limit of the sequence using L'Hôpital's Rule.

$$a_n = \frac{(\ln(n))^2}{9n}$$

(Use symbolic notation and fractions where needed. Enter DNE if the sequence diverges.)

Solution

Note that $a_n = f(n)$ for $f(x) = \frac{(\ln(x))^2}{9x}$.

Apply L'Hôpital's twice.

$$\lim_{x\to\infty} \frac{(\ln(x))^2}{9x} = \lim_{x\to\infty} \frac{\left((\ln(x))^2\right)'}{(9x)'}$$
$$= \lim_{x\to\infty} \frac{\frac{2\ln(x)}{x}}{9}$$
$$= \lim_{x\to\infty} \frac{2\ln(x)}{9x}$$
$$= \lim_{x\to\infty} \frac{(2\ln(x))'}{(9x)'}$$
$$= \lim_{x\to\infty} \frac{\frac{2}{x}}{9}$$
$$= \lim_{x\to\infty} \frac{2}{9x}$$
$$= 0$$

(EX) 10.1.61 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | DEFAULT FEEDBACK

Use the appropriate limit laws and theorems to determine the limit of the sequence.

$$a_n = \frac{n}{9}\sin\left(\frac{2}{n}\right)$$

(Use symbolic notation and fractions where needed. Enter DNE if the sequence diverges.)

Solution

Use the Theorem on Sequences Defined by a Function.

$$\lim_{n\to\infty} \frac{n}{9}\sin\left(\frac{2}{n}\right) = \lim_{x\to\infty} \frac{x}{9}\sin\left(\frac{2}{x}\right)$$

Now by L'Hôpital's Rule,

$$\lim_{x\to\infty} \frac{x}{9}\sin\left(\frac{2}{x}\right) = \frac{1}{9}\lim_{x\to\infty} \frac{\sin\left(\frac{2}{x}\right)}{\frac{1}{x}}$$
$$= \frac{1}{9}\lim_{x\to\infty} \frac{\left(\sin\left(\frac{2}{x}\right)\right)'}{\left(\frac{1}{x}\right)'}$$
$$= \frac{1}{9}\lim_{x\to\infty} \frac{\cos\left(\frac{2}{x}\right)\cdot\left(-\frac{2}{x^2}\right)}{-\frac{1}{x^2}}$$
$$= \frac{1}{9}\lim_{x\to\infty} \left(2\cos\left(\frac{2}{x}\right)\right)$$
$$= \frac{2}{9}\lim_{x\to\infty} \cos\left(\frac{2}{x}\right)$$
$$= \frac{2}{9}\cos(0)$$
$$= \frac{2}{9}$$

Thus,

$$\lim_{n\to\infty} \frac{n}{9}\sin\left(\frac{2}{n}\right) = \frac{2}{9}$$

(EX) 10.1.43 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | DEFAULT FEEDBACK

Use the appropriate limit laws and theorems to determine the limit of the sequence.

$$a_n = 3^{1/n}$$

(Use symbolic notation and fractions where needed. Enter DNE if the sequence diverges.)

Solution

Since $f(x) = 3^x$ is a continuous function and $f\left(\frac{1}{n}\right) = 3^{1/n} = a_n$, it follows that

$$\lim_{n\to\infty} 3^{1/n} = f\left(\lim_{n\to\infty} \frac{1}{n}\right)$$
$$= 3^{\lim_{x\to\infty} 1/x}$$
$$= 3^0$$
$$= 1$$

(EX) 10.1.44 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | DEFAULT FEEDBACK

Use the appropriate limit laws and theorems to determine the limit of the sequence.

$$b_n = n^{4/n}$$

(Use symbolic notation and fractions where needed. Enter DNE if the sequence diverges.)

Solution

Take the natural logarithm of both sides of the expression $b_n = n^{4/n}$ to obtain

$$\ln b_n = \ln\left(n^{4/n}\right) = \frac{4\ln(n)}{n}$$

Thus,

$$\lim_{n\to\infty} \ln b_n = \lim_{n\to\infty} \frac{4\ln(n)}{n}$$
$$= 4\lim_{x\to\infty} \frac{\ln(x)}{x}$$
$$= 4\lim_{x\to\infty} \frac{(\ln(x))'}{x'}$$
$$= 4\lim_{x\to\infty} \frac{1}{x}$$
$$= 0$$

Since $f(x) = e^x$ is a continuous function and $f(\ln(b_n)) = e^{\ln(b_n)} = b_n$, it follows that

$$\lim_{n\to\infty} b_n = f\left(\lim_{n\to\infty} \ln(b_n)\right)$$
$$= e^{\lim_{n\to\infty} \ln(b_n)}$$
$$= e^0$$
$$= 1$$

(EX) 10.1.66 (Rogawski 4e ET) Ro

QUESTION VIEW | SOLUTION | DEFAULT FEEDBACK

Use the appropriate limit laws and theorems to determine the limit of the sequence.

$$a_n = \left(1 + \frac{8}{n^2}\right)^n$$

(Use symbolic notation and fractions where needed. Enter DNE if the sequence diverges.)

Solution

Take the natural logarithm of both sides of the expression $a_n = \left(1 + \frac{8}{n^2}\right)^n$.

$$\ln(a_n) = \ln\left(1 + \frac{8}{n^2}\right)^n = n\ln\left(1 + \frac{8}{n^2}\right)$$

Therefore,

$$\begin{aligned}\lim_{n\to\infty} \ln(a_n) &= \lim_{n\to\infty} n\ln\left(1 + \frac{8}{n^2}\right)\\ &= \lim_{x\to\infty} x\ln\left(1 + \frac{8}{x^2}\right)\\ &= \lim_{x\to\infty} \frac{\ln\left(1 + \frac{8}{x^2}\right)}{\frac{1}{x}}\end{aligned}$$

By L'Hôpital's Rule,

$$\begin{aligned}\lim_{n\to\infty} \ln(a_n) &= \lim_{x\to\infty} \frac{\left(\ln\left(1 + \frac{8}{x^2}\right)\right)'}{\left(\frac{1}{x}\right)'}\\ &= \lim_{x\to\infty} \frac{\frac{1}{1+\frac{8}{x^2}} \cdot \left(-\frac{16}{x^3}\right)}{-\frac{1}{x^2}}\\ &= \lim_{x\to\infty} \frac{\frac{16}{x}}{1 + \frac{8}{x^2}}\\ &= \frac{0}{1+0}\\ &= 0\end{aligned}$$

Since $f(x) = e^x$ is a continuous function and $a_n = e^{\ln(a_n)} = f(\ln(a_n))$, it follows that

$$\begin{aligned}\lim_{n\to\infty} a_n &= f\left(\lim_{n\to\infty} \ln(a_n)\right)\\ &= e^{\lim_{n\to\infty} \ln(a_n)}\\ &= e^0\\ &= 1\end{aligned}$$

(EX) 10.1.65 (Rogawski 4e ET) Rog

QUESTION VIEW | SOLUTION | DEFAULT FEEDBACK

Use the appropriate limit laws and theorems to determine the limit of the sequence.

$$a_n = \left(1 + \frac{7}{n}\right)^n$$

(Use symbolic notation and fractions where needed. Enter DNE if the sequence diverges.)

Solution

Take the natural logarithm of both sides of the expression $a_n = \left(1 + \frac{7}{n}\right)^n$.

$$\ln(a_n) = \ln\left(1 + \frac{7}{n}\right)^n = n\ln\left(1 + \frac{7}{n}\right)$$

Therefore,

$$\begin{aligned}\lim_{n\to\infty} \ln a_n &= \lim_{n\to\infty} n\ln\left(1 + \frac{7}{n}\right)\\ &= \lim_{x\to\infty} x\ln\left(1 + \frac{7}{x}\right)\\ &= \lim_{x\to\infty} \frac{\ln\left(1 + \frac{7}{x}\right)}{\frac{1}{x}}\end{aligned}$$

By L'Hôpital's Rule,

$$\begin{aligned}\lim_{n\to\infty} \ln(a_n) &= \lim_{x\to\infty} \frac{\left(\ln\left(1 + \frac{7}{x}\right)\right)'}{\left(\frac{1}{x}\right)'}\\ &= \lim_{x\to\infty} \frac{\frac{1}{1+\frac{7}{x}} \cdot \left(-\frac{7}{x^2}\right)}{-\frac{1}{x^2}}\\ &= \lim_{x\to\infty} \frac{7}{1 + \frac{7}{x}}\\ &= \frac{7}{1+0}\\ &= 7\end{aligned}$$

Since $f(x) = e^x$ is a continuous function and $a_n = e^{\ln(a_n)} = f(\ln(a_n))$, it follows that

$$\begin{aligned}\lim_{n\to\infty} a_n &= f\left(\lim_{n\to\infty} \ln(a_n)\right)\\ &= e^{\lim_{n\to\infty} \ln(a_n)}\\ &= e^7\end{aligned}$$

(EX) 10.1.69 (Rogawski 4e ET) Rogaw

QUESTION VIEW | SOLUTION | DEFAULT FEEDBACK

Find the limit of the sequence using L'Hôpital's Rule.

$$c_n = n\left(\sqrt{n^2 + 9} - n\right)$$

(Use symbolic notation and fractions where needed. Enter DNE if the sequence diverges.)

Solution

Note that $a_n = f(n)$ for $f(x) = x\left(\sqrt{x^2 + 9} - x\right)$.

Multiply and divide by $\left(\sqrt{x^2 + 9} + x\right)$ and apply L'Hôpital's Rule.

$$\begin{aligned}\lim_{x\to\infty} x\left(\sqrt{x^2 + 9} - x\right) &= \lim_{x\to\infty} \frac{x\left(\sqrt{x^2 + 9} - x\right)\left(\sqrt{x^2 + 9} + x\right)}{\sqrt{x^2 + 9} + x}\\ &= \lim_{x\to\infty} \frac{9x}{\sqrt{x^2 + 9} + x}\\ &= \lim_{x\to\infty} \frac{(9x)'}{\left(\sqrt{x^2 + 9} + x\right)'}\\ &= \lim_{x\to\infty} \frac{9}{\frac{x}{\sqrt{x^2+9}} + 1}\\ &= \lim_{x\to\infty} \frac{9}{\sqrt{\frac{x^2}{x^2+9}} + 1}\\ &= \lim_{x\to\infty} \frac{9}{\sqrt{\frac{1}{1+9/x^2}} + 1}\\ &= \frac{9}{2}\end{aligned}$$

(EX) 10.2.18 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | INCORRECT FEEDBACK | DEFAULT FEEDBACK

Calculate the limit.

$$\lim_{n\to\infty} \frac{n}{\sqrt{n^2+13}}$$

Solution

To calculate the limit, begin by rewriting the rational expression so that the entire term is under the radical.

$$\frac{n}{\sqrt{n^2+13}} = \sqrt{\frac{n^2}{n^2+13}}$$

Next, divide each term in the numerator and denominator of the radicand by n^2. Then evaluate the limit.

$$\lim_{n\to\infty} \frac{n}{\sqrt{n^2+13}} = \lim_{n\to\infty} \sqrt{\frac{n^2}{n^2+13}}$$

$$= \lim_{n\to\infty} \sqrt{\frac{1}{1+\left(\frac{13}{n^2}\right)}}$$

$$= 1$$

Therefore, the given limit equals 1.

The nth Term Divergence Test states if $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges.

Because $\lim_{n\to\infty} \frac{n}{\sqrt{n^2+13}} = 1 \neq 0$, then the series $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+13}}$ diverges.

(EX) 10.2.20 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | INCORRECT FEEDBACK | DEFAULT FEEDBACK

Calculate the limit.

$$\lim_{n\to\infty} (-1)^n n^5$$

Solution

The general term $a_n = (-1)^n n^5$ does not tend to zero. In fact, $\lim_{n\to\infty} n^5 = \infty$, so the odd terms a_{2n+1} tend to $-\infty$, and the even terms a_{2n} tend to ∞. Therefore, $\lim_{n\to\infty} a_n$ does not exist.

By the nth Term Divergence Test, because $\lim_{n\to\infty} a_n$ does not exist, the series $\sum_{n=1}^{\infty} (-1)^n n^5$ diverges.

(EX) 10.2.17 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | INCORRECT FEEDBACK | DEFAULT FEEDBACK

Calculate the limit.

$$\lim_{n\to\infty} \frac{n}{11n+20}$$

Solution

First, divide both the numerator and denominator of the limit by n. Then evaluate the limit.

$$\lim_{n\to\infty} \frac{n}{11n+20} = \lim_{n\to\infty} \frac{1}{11+(20/n)}$$

$$= \frac{1}{11}$$

Thus, the limit equals $\frac{1}{11}$.

By the nth Term Divergence Test, as $\lim_{n\to\infty} a_n \neq 0$, where $a_n = \frac{n}{11n+20}$, the series $\sum_{n=1}^{\infty} \frac{n}{11n+20}$ diverges.

(EX) 10.2.11 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | INCORRECT FEEDBACK - 1 | DEFAULT FEEDBACK

Compute the partial sums S_3, S_4, and S_5 for the series and then find its sum.

$$\sum_{n=1}^{\infty} \left(\frac{1}{n+2} - \frac{1}{n+3}\right)$$

(Use symbolic notation and fractions where needed.)

Solution

Compute the partial sums S_3, S_4, and S_5.

$$S_3 = \left(\frac{1}{3}-\frac{1}{4}\right) + \left(\frac{1}{4}-\frac{1}{5}\right) + \left(\frac{1}{5}-\frac{1}{6}\right)$$

$$= \frac{1}{3} - \frac{1}{6}$$

$$= \frac{1}{6}$$

$$S_4 = S_3 + \left(\frac{1}{6}-\frac{1}{7}\right)$$

$$= \frac{1}{6} + \frac{1}{42}$$

$$= \frac{4}{21}$$

$$S_5 = S_4 + \left(\frac{1}{7}-\frac{1}{8}\right)$$

$$= \frac{4}{21} + \frac{1}{56}$$

$$= \frac{5}{24}$$

The general term in the sequence of partial sums is

$$S_N = \left(\frac{1}{3}-\frac{1}{4}\right) + \left(\frac{1}{4}-\frac{1}{5}\right) + \left(\frac{1}{5}-\frac{1}{6}\right) + \ldots + \left(\frac{1}{N+2}-\frac{1}{N+3}\right) = \frac{1}{3} - \frac{1}{N+3}$$

Thus,

$$S = \lim_{N\to\infty} S_N = \lim_{N\to\infty} \left(\frac{1}{3} - \frac{1}{N+3}\right) = \frac{1}{3}$$

Therefore, the sum of the series is $\frac{1}{3}$.

(EX) 10.2.13 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | INCORRECT FEEDBACK - 1 | DEFAULT FEEDBACK

Calculate S_3, S_4, and S_5 and then find the sum $\sum_{n=1}^{\infty} \frac{1}{4n^2-9}$ using the identity

$$\frac{1}{4n^2-9} = \frac{1}{6}\left(\frac{1}{2n-3} - \frac{1}{2n+3}\right)$$

Solution

$$S_3 = \sum_{n=1}^{3} \frac{1}{4n^2-9} = \frac{1}{6}\sum_{n=1}^{3}\left(\frac{1}{2n-3} - \frac{1}{2n+3}\right)$$

$$= \frac{1}{6}\left[\left(\frac{1}{2(1)-3} - \frac{1}{2(1)+3}\right) + \left(\frac{1}{2(2)-3} - \frac{1}{2(2)+3}\right) + \left(\frac{1}{2(3)-3} - \frac{1}{2(3)+3}\right)\right]$$

$$= \frac{1}{6}\left[\left(\frac{1}{-1} - \frac{1}{5}\right) + \left(\frac{1}{1} - \frac{1}{7}\right) + \left(\frac{1}{3} - \frac{1}{9}\right)\right]$$

$$= \frac{1}{6}\left[\left(\frac{-6}{5}\right) + \left(\frac{-6}{-7}\right) + \left(\frac{-6}{-27}\right)\right]$$

$$= -\left[\left(\frac{1}{5}\right) + \left(\frac{1}{-7}\right) + \left(\frac{1}{-27}\right)\right] = -\frac{19}{945}$$

$$S_4 = S_3 + \frac{1}{2}\left(\frac{1}{2(4)-3} - \frac{1}{2(4)+3}\right)$$

$$= -\frac{19}{945} + \frac{1}{6}\left(\frac{1}{5} - \frac{1}{11}\right) = -\frac{4}{2079}$$

$$S_5 = S_4 + \frac{1}{2}\left(\frac{1}{2(5)-3} - \frac{1}{2(5)+3}\right)$$

$$= -\frac{4}{2079} + \frac{1}{6}\left(\frac{1}{7} - \frac{1}{13}\right) = \frac{35}{3861}$$

The general term in the sequence of partial sums for the series on the right–hand side is

$$S_N = \frac{1}{6}\left[\left(\cancel{\frac{1}{-1}} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{1}} - \cancel{\frac{1}{7}}\right) + \left(\frac{1}{3} - \cancel{\frac{1}{9}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{11}}\right) + \left(\cancel{\frac{1}{7}} - \cancel{\frac{1}{13}}\right)\right.$$

$$+ \left(\cancel{\frac{1}{9}} - \cancel{\frac{1}{15}}\right) + \cdots + \left(\cancel{\frac{1}{2N-9}} - \cancel{\frac{1}{2N-3}}\right) + \left(\cancel{\frac{1}{2N-7}} - \frac{1}{2N-1}\right)$$

$$+ \left(\cancel{\frac{1}{2N-5}} - \frac{1}{2N+1}\right) + \left(\cancel{\frac{1}{2N-3}} - \frac{1}{2N+3}\right)$$

$$= \frac{1}{6}\left(\frac{1}{3} - \frac{1}{2N-1} - \frac{1}{2N+1} - \frac{1}{2N+3}\right)$$

$$= \frac{1}{6}\left(\frac{1}{3} - \frac{1}{2N-1} - \frac{1}{2N+1} - \frac{1}{2N+3}\right)$$

Finally, find the sum.

$$\sum_{n=1}^{\infty} \frac{1}{4n^2-9} = \lim_{N\to\infty}\left[\frac{1}{6}\left(\frac{1}{3} - \frac{1}{2N-1} - \frac{1}{2N+1} - \frac{1}{2N+3}\right)\right]$$

$$= \frac{1}{18}$$

(EX) 10.2.05 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | INCORRECT FEEDBACK - 1 | DEFAULT FEEDBACK

Compute the partial sums S_2, S_4, and S_6 of

$$\frac{1}{4\cdot 7} + \frac{1}{7\cdot 10} + \frac{1}{10\cdot 13} + \frac{1}{13\cdot 16} + \cdots$$

(Use symbolic notation and fractions where needed.)

Solution

Compute the partial sums.

$$S_2 = \frac{1}{4\cdot 7} + \frac{1}{7\cdot 10}$$
$$= \frac{1}{28} + \frac{1}{70}$$
$$= \frac{1}{20}$$

$$S_4 = S_2 + \frac{1}{10\cdot 13} + \frac{1}{13\cdot 16}$$
$$= \frac{1}{20} + \frac{1}{130} + \frac{1}{208}$$
$$= \frac{1}{16}$$

Notice that the 5th and the 6th terms are $a_5 = \frac{1}{16\cdot 19}$ and $a_6 = \frac{1}{19\cdot 22}$. Therefore,

$$S_6 = S_4 + \frac{1}{16\cdot 19} + \frac{1}{19\cdot 22}$$
$$= \frac{1}{16} + \frac{1}{304} + \frac{1}{418}$$
$$= \frac{3}{44}$$

(EX) 10.2.15 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | DEFAULT FEEDBACK

Find the sum of the infinite series.

$$\frac{1}{4\cdot 16} + \frac{1}{16\cdot 28} + \frac{1}{28\cdot 40} + \frac{1}{40\cdot 52} + \ldots$$

(Use symbolic notation and fractions where needed.)

Solution

First, rewrite this sum.

$$\sum_{n=1}^{\infty} \frac{1}{(12n-8)(12n+4)} = \sum_{n=1}^{\infty} \frac{1}{12}\left(\frac{1}{12n-8} - \frac{1}{12n+4}\right)$$

Then find the general term in the sequence of partial sums

$$S_N = \frac{1}{12}\left(\frac{1}{4} - \frac{1}{16}\right) + \frac{1}{12}\left(\frac{1}{16} - \frac{1}{28}\right)$$
$$+ \frac{1}{12}\left(\frac{1}{28} - \frac{1}{40}\right) + \ldots + \frac{1}{12}\left(\frac{1}{12N-8} - \frac{1}{12N+4}\right)$$
$$= \frac{1}{12}\left(\frac{1}{4} - \frac{1}{12N+4}\right)$$

Thus,

$$\lim_{N\to\infty} S_N = \lim_{N\to\infty} \frac{1}{12}\left(\frac{1}{4} - \frac{1}{12N+4}\right) = \frac{1}{48}$$

Finally, find the sum.

$$\sum_{n=1}^{\infty} \frac{1}{(12n-8)(12n+4)} = \frac{1}{48}$$

(EX) 10.2.60 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | DEFAULT FEEDBACK

Evaluate $\sum_{n=1}^{\infty} \frac{3}{(n+6)(n+7)(n+8)}$.

(Use symbolic notation and fractions where needed.)

$$\sum_{n=1}^{\infty} \frac{3}{(n+6)(n+7)(n+8)} = \frac{3}{112}$$

Solution

First, find constants A, B, C such that

$$\frac{3}{(n+6)(n+7)(n+8)} = \frac{A}{n+6} + \frac{B}{n+7} + \frac{C}{n+8}$$

Clear denominators.

$$3 = A(n+7)(n+8) + B(n+6)(n+8) + C(n+6)(n+7)$$

Setting $n = -6$ now yields $A = \frac{3}{2}$, while setting $n = -7$ yields $B = -3$ and setting $n = -8$ yields $C = \frac{3}{2}$.

Thus,

$$\frac{3}{(n+6)(n+7)(n+8)} = \frac{3}{2}\left(\frac{1}{n+6} - \frac{2}{n+7} + \frac{1}{n+8}\right)$$

So, the sum is

$$\sum_{n=1}^{\infty} \frac{3}{(n+6)(n+7)(n+8)} = \sum_{n=1}^{\infty} \frac{3}{2}\left(\frac{1}{n+6} - \frac{2}{n+7} + \frac{1}{n+8}\right)$$

The general term of the sequence of partial sums for the series on the right-hand side is

$$S_N = \frac{3}{2}\left(\frac{1}{56} - \frac{1}{N+7} + \frac{1}{N+8}\right)$$

Thus,

$$\sum_{n=1}^{\infty} \frac{3}{(n+6)(n+7)(n+8)} = \lim_{N\to\infty} S_N = \lim_{N\to\infty} \frac{3}{2}\left(\frac{1}{56} - \frac{1}{N+7} + \frac{1}{N+8}\right) = \frac{3}{112}$$

(EX) 10.2.52 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | INCORRECT FEEDBACK - 1 | DEFAULT FEEDBACK

Suppose that $\sum_{n=1}^{\infty} a_n$ is an infinite series with partial sum $S_N = 10 - \frac{2}{N^2}$.

What are the values of $\sum_{n=1}^{10} a_n$ and $\sum_{n=5}^{16} a_n$?

Solution

Find the sum of the first ten terms of the series.

$$\sum_{n=1}^{10} a_n = S_{10} = 10 - \frac{2}{10^2} = \frac{499}{50}$$

Thus, the value of $\sum_{n=1}^{10} a_n$ is $\frac{499}{50}$.

To determine the sum of terms from the fifth to the sixteenth, find the difference between the sum of the first sixteen terms and the sum of the first four terms.

$$\sum_{n=5}^{16} a_n = S_{16} - S_4 = \left(10 - \frac{2}{16^2}\right) - \left(10 - \frac{2}{4^2}\right) = \frac{2}{4^2} - \frac{2}{16^2} = \frac{15}{128}$$

Thus, the value of $\sum_{n=5}^{16} a_n$ is $\frac{15}{128}$.

The value of a_3 is given by the difference between S_3 and S_2. Therefore,

$$a_3 = S_3 - S_2 = \left(10 - \frac{2}{3^2}\right) - \left(10 - \frac{2}{2^2}\right) = \frac{2}{2^2} - \frac{2}{3^2} = \frac{5}{18}$$

Thus, the value of a_3 is $\frac{5}{18}$.

The general formula for a_n can be found as the difference between S_n and S_{n-1}.

$$a_n = S_n - S_{n-1}$$

$$= \left(10 - \frac{2}{n^2}\right) - \left(10 - \frac{2}{(n-1)^2}\right)$$

$$= \frac{2}{(n-1)^2} - \frac{2}{n^2}$$

$$= \frac{2\left(n^2 - (n-1)^2\right)}{(n(n-1))^2}$$

$$= \frac{2\left(n^2 - n^2 + 2n - 1\right)}{(n(n-1))^2}$$

$$= \frac{2(2n-1)}{(n(n-1))^2}$$

Thus, the general formula for a_n is $\frac{2(2n-1)}{(n(n-1))^2}$.

The sum $\sum_{n=1}^{\infty} a_n$ is the limit of the sequence of partial sums $\{S_N\}$. Hence,

$$\sum_{n=1}^{\infty} a_n = \lim_{N\to\infty} S_N = \lim_{N\to\infty}\left(10 - \frac{2}{N^2}\right) = 10$$

Thus, the sum $\sum_{n=1}^{\infty} a_n$ is equal to 10.

(EX) 10.2.47 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | INCORRECT FEEDBACK - 1 | DEFAULT FEEDBACK

Which of the following are geometric series?

- [ ] $\sum_{n=0}^{\infty} \frac{3^n}{29^n}$
- [ ] $\sum_{n=0}^{\infty} \frac{3}{n^5}$
- [ ] $\sum_{n=0}^{\infty} \frac{n^3}{3^n}$
- [ ] $\sum_{n=0}^{\infty} (3\pi)^{-n}$

This ratio is not constant because it depends on n. Hence, the series $\sum_{n=0}^{\infty} \frac{3}{n^5}$ is not a geometric series.

Next, consider the series $\sum_{n=0}^{\infty} \frac{n^3}{3^n}$. The ratio between two successive terms is

$$\frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)^3}{3^{n+1}}}{\frac{n^3}{3^n}}$$

$$= \frac{(n+1)^3}{n^3} \cdot \frac{3^n}{3^{n+1}}$$

Solution

Rewrite the series $\sum_{n=0}^{\infty} \frac{3^n}{29^n}$ as follows.

$$\sum_{n=0}^{\infty} \frac{3^n}{29^n} = \sum_{n=0}^{\infty} \left(\frac{3}{29}\right)^n$$

Therefore, this series is a geometric series with the first term $c = 1$ and with the common ratio $r = \frac{3}{29}$.

Next, consider the series $\sum_{n=0}^{\infty} \frac{3}{n^5}$. The ratio between two successive terms is

$$\frac{a_{n+1}}{a_n} = \frac{\frac{3}{(n+1)^5}}{\frac{3}{n^5}}$$

$$= \frac{n^5}{(n+1)^5}$$

$$= \left(\frac{n}{n+1}\right)^5$$

This ratio is not constant because it depends on n. Hence, the series $\sum_{n=0}^{\infty} \frac{n^3}{3^n}$ is not a geometric series.

Rewrite the series $\sum_{n=0}^{\infty} (3\pi)^{-n}$ as follows.

$$\sum_{n=0}^{\infty} (3\pi)^{-n} = \sum_{n=0}^{\infty} \left(\frac{1}{3\pi}\right)^n$$

Therefore, this series is a geometric series with the first term $c = 1$ and with the common ratio $r = \frac{1}{3\pi}$.

(EX) 10.2.16 (Rogawski 4e ET)

Rogawski 4e Calc

QUESTION VIEW | SOLUTION | INCORRECT FEEDBACK | DEFAULT FEEDBACK

Consider the infinite series $\sum_{n=1}^{\infty} (-1)^{n-1}$ and determine the following.

Find the formula for the partial sum S_N of the series.

- ○ $S_N = 1$
- ○ $S_N = -1$
- ○ $S_N = 0$
- ○ $S_N = \begin{cases} 1 & \text{if } N \text{ is even} \\ 0 & \text{if } N \text{ is odd} \end{cases}$
- ○ $S_N = \begin{cases} 1 & \text{if } N \text{ is odd} \\ 0 & \text{if } N \text{ is even} \end{cases}$
- ○ $S_N = \begin{cases} 1 & \text{if } N \text{ is even} \\ -1 & \text{if } N \text{ is odd} \end{cases}$

Solution

The partial sums of the series are:

$S_1 = (-1)^{1-1} = 1;$

$S_2 = (-1)^0 + (-1)^1 = 1 - 1 = 0;$

$S_3 = (-1)^0 + (-1)^1 + (-1)^2 = 1 - 1 + 1 = 1;$

$S_4 = (-1)^0 + (-1)^1 + (-1)^2 + (-1)^3 = 1 - 1 + 1 - 1 = 0;$

In general, observe that when N is odd, the partial sum is 1 and when N is even the partial sum is 0, therefore,

$$S_N = \begin{cases} 1 & \text{if } N \text{ is odd} \\ 0 & \text{if } N \text{ is even} \end{cases}$$

Because the values of S_N alternate between 0 and 1, the sequence of partial sums diverges; this, in turn, implies that the series $\sum_{n=1}^{\infty} (-1)^{n-1}$ diverges.

(EX) 10.2.25 (Rogawski 4e ET)

Rogaw

QUESTION VIEW | SOLUTION | INCORRECT FEEDBACK - 1 | DEFAULT FEEDBACK

Consider the series.

$$\frac{14}{3} + \frac{14}{3^2} + \frac{14}{3^3} + \frac{14}{3^4} + \cdots$$

Solution

First, rewrite this sum.

$$\frac{14}{3} + \frac{14}{3^2} + \frac{14}{3^3} + \frac{14}{3^4} + \cdots = \sum_{n=0}^{\infty} \frac{14}{3}\left(\frac{1}{3}\right)^n$$

This is a geometric series with the first term $c = \frac{14}{3}$ and with the common ratio $r = \frac{1}{3}$. Because $0 < r < 1$, use the formula for the sum of a geometric series.

$$\sum_{n=0}^{\infty} \frac{14}{3}\left(\frac{1}{3}\right)^n = \frac{c}{1-r}$$

$$= \frac{\frac{14}{3}}{1 - \frac{1}{3}}$$

$$= 7$$

Thus, the sum of the geometric series is 7.

(EX) 10.2.26 (Rogawski 4e ET) (ESF)

Rogawski 4e Calcu

QUESTION VIEW | SOLUTION | INCORRECT FEEDBACK - 1 | DEFAULT FEEDBACK

Consider the series.

$$\left(\frac{11}{3}\right)+\left(\frac{11}{3}\right)^2+\left(\frac{11}{3}\right)^3+\left(\frac{11}{3}\right)^4+\cdots$$

This series can be written as a geometric series in the form $\sum_{n=0}^{\infty} cr^n$. Identify c and r in the geometric series.

Solution

Write the given series in the form $\sum_{n=0}^{\infty} cr^n$. Notice that the summation starts with $n = 0$ even though the exponent within the series starts at 1.

$$\left(\frac{11}{3}\right)+\left(\frac{11}{3}\right)^2+\left(\frac{11}{3}\right)^3+\left(\frac{11}{3}\right)^4+\cdots=\sum_{n=1}^{\infty}\left(\frac{11}{3}\right)^n$$

$$=\sum_{n=0}^{\infty}\left(\frac{11}{3}\right)^{n+1}$$

$$=\sum_{n=0}^{\infty}\left(\frac{11}{3}\right)\cdot\left(\frac{11}{3}\right)^n$$

Therefore, the given series is a geometric series with $c = \frac{11}{3}$ and the common ratio $r = \frac{11}{3}$.

Because $|r| = \frac{11}{3} > 1$, the series diverges.

(EX) 10.2.28 (Rogawski 4e ET) (ESF)

R

QUESTION VIEW | SOLUTION | INCORRECT FEEDBACK - 1 | DEFAULT FEEDBACK

Use the formula for the sum of a geometric series to find the sum.

(Use symbolic notation and fractions where needed. Enter DNE if the series diverges.)

$$\sum_{n=2}^{\infty}\frac{7\cdot(-3)^n}{4^n} = \frac{9}{4}$$

Solution

This is a geometric series with first term $c = 7\left(-\frac{3}{4}\right)^2$ and with common ratio $r = -\frac{3}{4}$.

So, using the formula for the sum of a geometric series gives,

$$\frac{c}{1-r}=\frac{7\cdot\left(\frac{9}{16}\right)}{1+\frac{3}{4}}=\frac{63}{16}\cdot\frac{4}{7}=\frac{9}{4}$$

(EX) 10.2.36 (Rogawski 4e ET) (ESF)

Rogawski 4e Calculus Early Transce

QUESTION VIEW | SOLUTION | INCORRECT FEEDBACK - 1 | DEFAULT FEEDBACK

Use the formula for the sum of a geometric series to calculate the given sum.

(Express numbers in exact form. Use symbolic notation and fractions where needed. Enter DNE if the series diverges.)

$$\frac{2^2}{3^5}+\frac{2^3}{3^6}+\frac{2^4}{3^7}+\frac{2^5}{3^8}+\cdots = \frac{4}{81}$$

Solution

First, rewrite this sum.

$$\frac{2^2}{3^5}+\frac{2^3}{3^6}+\frac{2^4}{3^7}+\frac{2^5}{3^8}+\cdots=\sum_{n=0}^{\infty}\frac{4}{243}\left(\frac{2}{3}\right)^n$$

This is a geometric series with first term $c = \frac{2^2}{3^5} = \frac{4}{243}$ and with common ratio $r = \frac{2}{3}$. Because $0 < r < 1$, use the formula for the sum of a geometric series.

$$\sum_{n=0}^{\infty}\frac{4}{243}\left(\frac{2}{3}\right)^n=\frac{\frac{4}{243}}{1-\frac{2}{3}}=\frac{\frac{4}{243}}{\frac{1}{3}}=\frac{4}{81}$$

(EX) 10.2.38 (Rogawski 4e ET)

R

QUESTION VIEW | SOLUTION | DEFAULT FEEDBACK

Use the formula for the sum of a geometric series to find the sum.

(Use symbolic notation and fractions where needed. Enter DNE if the series diverges.)

$$\frac{64}{49}+\frac{8}{7}+1+\frac{7}{8}+\frac{49}{64}+\cdots = \frac{512}{49}$$

Solution

First, rewrite this sum.

$$\frac{64}{49}+\frac{8}{7}+1+\frac{7}{8}+\frac{49}{64}+\cdots=\sum_{n=-2}^{\infty}\left(\frac{7}{8}\right)^n$$

This is a geometric series with first term $c = \frac{64}{49}$ and with common ratio $r = \frac{7}{8}$. Since $0 < r < 1$, thus, using the formula fo the sum of a geometric series yields

$$\sum_{n=-2}^{\infty}\left(\frac{7}{8}\right)^n=\frac{\frac{64}{49}}{1-\frac{7}{8}}=\frac{\frac{64}{49}}{\frac{1}{8}}=\frac{512}{49}$$

(EX) 10.2.32 (Rogawski 4e ET)

R

QUESTION VIEW | SOLUTION | DEFAULT FEEDBACK

Use the formula for the sum of a geometric series to find the sum.

(Use symbolic notation and fractions where needed. Enter DNE if the series diverges.)

$$\sum_{n=7}^{\infty} e^{7-2n} = \frac{e^{-5}}{e^2-1}$$

Solution

First, rewrite the series.

$$\sum_{n=7}^{\infty} e^{7-2n}=\sum_{n=7}^{\infty} e^7e^{-2n}=\sum_{n=7}^{\infty} e^7\left(\frac{1}{e^2}\right)^n$$

This is a geometric series with first term $c = e^7\left(\frac{1}{e^2}\right)^7 = e^{-7}$ and with common ratio $r = \frac{1}{e^2}$.

Since $0 < r < 1$, thus, using the formula for the sum of a geometric series gives,

$$\sum_{n=7}^{\infty} e^{7-2n}=e^{-7}\frac{1}{1-\frac{1}{e^2}}=e^{-7}\frac{e^2}{e^2-1}=\frac{e^{-5}}{e^2-1}$$

(EX) 10.2.27 (Rogawski 4e)

QUESTION VIEW | SOLUTION | DEFAULT FEEDBACK

Use the formula for the sum of a geometric series to find the sum.

(Use symbolic notation and fractions where needed. Enter DNE if the series diverges.)

$$\sum_{n=2}^{\infty}\left(\frac{2}{13}\right)^{-n} = \text{DNE}$$

Solution

First, rewrite the series as

$$\left(\frac{2}{13}\right)^{-n}=\left(\frac{13}{2}\right)^n$$

This is a geometric series with common ratio $r = \frac{13}{2}$. Since $\frac{13}{2} > 1$, this is a geometric series with $r > 1$, so it diverges.

(EX) 10.2.44 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | DEFAULT FEEDBACK

Identify a reduced fraction that has the decimal expansion $0.303333333333\ldots$

(Give an exact answer. Use symbolic notation and fractions as needed.)

$0.303333333333\ldots = \frac{91}{300}$

Solution

The decimal $0.303333333333\ldots$ may be regarded as a constant plus a geometric series.

$$0.303333333333\ldots = 0.30 + 0.003 + 0.0003 + 0.00003\ldots$$

$$= \frac{3}{10} + \frac{3}{1000} + \frac{3}{10000} + \frac{3}{100000} + \ldots$$

$$= \frac{3}{10} + \sum_{n=1}^{\infty} \frac{3}{10^{n+2}}$$

The second term is a geometric series with the first term $c = \frac{3}{1000}$ and with the common ratio $r = \frac{1}{10}$. Because $0 < r < 1$ use the formula for the sum of a geometric series.

$$\frac{3}{10} + \sum_{n=1}^{\infty} \frac{3}{10^{n+2}} = \frac{3}{10} + \frac{c}{1-r}$$

$$= \frac{3}{10} + \frac{\frac{3}{1000}}{1-\frac{1}{10}}$$

$$= \frac{3}{10} + \frac{3}{900}$$

$$= \frac{273}{900}$$

$$= \frac{91}{300}$$

Thus, the decimal $0.303333333333\ldots$ is equivalent to $\frac{91}{300}$.

(EX) 10.2.42 (Rogawski 4e ET)

Roga

QUESTION VIEW | SOLUTION | DEFAULT FEEDBACK

Determine a reduced fraction that has this repeating decimal.

(Use symbolic notation and fractions where needed.)

$0.107107107\cdots = \frac{107}{999}$

Solution

Sum of a Geometric Series

Let $c \neq 0$. If $|r| < 1$, then

$$\sum_{n=0}^{\infty} cr^n = c + cr + cr^2 + cr^3 + \cdots = \frac{c}{1-r}$$

First, write 0.107107107... decimal as the series.

$$0.107107107\cdots = \frac{107}{10^3} + \frac{107}{10^6} + \frac{107}{10^9} + \cdots = \sum_{n=1}^{\infty} \frac{107}{10^{3n}}$$

This is a geometric series with $c = \frac{107}{10^3}$ and $r = \frac{1}{10^3}$. Therefore, it converges to

$$0.107107107\cdots = \frac{\frac{107}{10^3}}{1-\frac{1}{10^3}} = \frac{107}{999}$$

(EX) 10.2.62 (Rogawski 4e ET) (ESF)

Rogawski 4e Calculus Early Transcend

ID: 2

UESTION VIEW | SOLUTION | INCORRECT FEEDBACK - 1 | DEFAULT FEEDBACK

A ball dropped from a height of 13 ft begins to bounce vertically. Each time it strikes the ground, it returns to two-thirds of its previous height. What is the total vertical distance traveled by the ball if it bounces infinitely many times?

(Give your answer as a whole number.)

Solution

If the ball is dropped from a height of h ft, the heights it returns each time are

$$\frac{2}{3}h, \quad \frac{2}{3}\cdot\frac{2}{3}h = \left(\frac{2}{3}\right)^2 h, \quad \frac{2}{3}\cdot\left(\frac{2}{3}\right)^2 h = \left(\frac{2}{3}\right)^3 h, \ldots$$

Each of these distances it travels twice, when going up and then when going down.

Therefore, the total distance d traveled by the ball is given by the following infinite sum.

$$d = h + 2\cdot\frac{2}{3}h + 2\cdot\left(\frac{2}{3}\right)^2 h + 2\cdot\left(\frac{2}{3}\right)^3 h + \cdots$$

$$= 2h - h + 2\cdot\frac{2}{3}h + 2\cdot\left(\frac{2}{3}\right)^2 h + 2\cdot\left(\frac{2}{3}\right)^3 h + \cdots$$

$$= 2h\left(1 + \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \cdots\right) - h$$

$$= 2h\sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^n - h$$

Use the formula for the sum of a geometric series to compute the sum of the resulting series and substitute $h = 13$ ft.

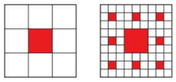
$$d = 2h\cdot\frac{1}{1-\frac{2}{3}} - h$$

$$= 5h$$

$$= 65 \text{ ft}$$

Thus, the total vertical distance traveled by the ball is 65 ft.

A unit square is cut into nine equal regions as in Figure (A). The central subsquare is painted red. Each of the unpainted squares is then cut into nine equal subsquares and the central square of each is painted red as in Figure (B). This procedure is repeated for each of the resulting unpainted squares.



(A) (B)

Solution

Initially, $\frac{1}{9}$ of the square is painted. Of the remaining unpainted $\frac{8}{9}$, another $\frac{1}{9}$ is painted, leaving $\frac{64}{81} = \left(\frac{8}{9}\right)^2$ unpainted. Another $\frac{1}{9}$ of that is painted in the third step. So eventually, the amount painted is $\sum_{n=0}^{\infty} \frac{1}{9}\left(\frac{8}{9}\right)^n$.

Then use the theorem of **Sum of a Geometric Series**.

Let $c \neq 0$. If $|r| < 1$, then

$$\sum_{n=0}^{\infty} cr^n = c + cr + cr^2 + cr^3 + \cdots = \frac{c}{1-r}$$

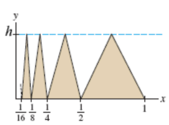
So, for $c = \frac{1}{9}$ and $r = \frac{8}{9}$

So, for $c = \frac{1}{9}$ and $r = \frac{8}{9}$

$$\sum_{n=0}^{\infty} \frac{1}{9}\left(\frac{8}{9}\right)^n = \frac{\frac{1}{9}}{1-\frac{8}{9}} = 1$$

Therefore, in the limit, the entire square is painted.

Compute the total area A of the (infinitely many) triangles in the figure, if the height h of each triangle is 33.



Solution

The area of a triangle with base B and height H is $A = \frac{1}{2}BH$. Because all of the triangles have height 33 the area of each triangle equals $A = \frac{33}{2}B$.

Now, for $n \geq 0$, the nth triangle has a base which extends from $x = \frac{1}{2^{n+1}}$ to $x = \frac{1}{2^n}$. Thus, the length of the base of one triangle is

$$B = \frac{1}{2^n} - \frac{1}{2^{n+1}} = \frac{1}{2^{n+1}}$$

Using this value of B, the equation of the area of one triangle is

$$A = \frac{33}{2}B = \frac{33}{2^{n+2}}$$

Then, the total area of the triangles is given by the geometric series $\sum_{n=0}^{\infty} \frac{33}{2^{n+2}}$.

To evaluate the total area, use the theorem of **Sum of a Geometric Series**.

Let $c \neq 0$. If $|r| < 1$, then

$$\sum_{n=0}^{\infty} cr^n = c + cr + cr^2 + cr^3 + \cdots = \frac{c}{1-r}$$

For the series $\sum_{n=0}^{\infty} \frac{33}{2^{n+2}}$, $c = \frac{33}{4}$ and $r = \frac{1}{2}$. Therefore,

$$\sum_{n=0}^{\infty} \frac{33}{2^{n+2}} = \sum_{n=0}^{\infty} \frac{33}{4}\left(\frac{1}{2}\right)^n = \frac{\frac{33}{4}}{1-\frac{1}{2}} = \frac{33}{2}$$

(EX) 10.3.04 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | INCORRECT FEEDBACK | DEFAULT FEEDBACK

Evaluate the integral

$$\int_6^\infty \frac{dx}{\sqrt{x-5}}$$

Solution

Rewrite the integral as a limit of a proper integral and evaluate it.

$$\begin{aligned}\int_6^\infty \frac{dx}{\sqrt{x-5}} &= \lim_{R\to\infty} \int_6^R \frac{dx}{\sqrt{x-5}} \\ &= 2\lim_{R\to\infty} \left(\sqrt{x-5}\,\Big|_6^R\right) \\ &= 2\lim_{R\to\infty} \left(\sqrt{R-5}-1\right) \\ &= \infty\end{aligned}$$

The function $f(x) = \frac{1}{\sqrt{x-5}}$ is continuous, positive, and decreasing on the interval $x \geq 6$, so the Integral Test can be applied.

The integral diverges; therefore, the series $\sum_{n=6}^{\infty} \frac{1}{\sqrt{n-5}}$ also diverges.

(EX) 10.3.02 (Rogawski 4e ET)

Rogawski 4e Calculus Early Tra

QUESTION VIEW | SOLUTION | INCORRECT FEEDBACK | DEFAULT FEEDBACK

Evaluate the integral

$$\int_1^\infty \frac{dx}{x+4}$$

(Express numbers in exact form. Use symbolic notation and fractions where needed. Use the symbol ∞ for infinity.)

Solution

Rewrite the integral as a limit of a proper integral and evaluate it.

$$\begin{aligned}\int_1^\infty \frac{dx}{x+4} &= \lim_{R\to\infty} \int_1^R \frac{dx}{x+4} \\ &= \lim_{R\to\infty} \left(\ln(|x+4|)\Big|_1^R\right) \\ &= \lim_{R\to\infty} \left(\ln(|R+4|) - \ln(5)\right) \\ &= \infty\end{aligned}$$

The function $f(x) = \frac{1}{x+4}$ is continuous, positive, and decreasing on the interval $x \geq 1$, so the Integral Test can be applied.

The integral diverges; therefore, the series $\sum_{n=1}^{\infty} \frac{1}{n+4}$ also diverges.

(EX) 10.3.77 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | DEFAULT FEEDBACK

Determine convergence or divergence of $\sum_{n=4}^{\infty} \frac{1}{n(\ln(n))^a}$ if $a = \frac{6}{5}$.

- (●) The series converges.
- ( ) The series diverges.

Solution

First consider the case $a > 0$ but $a \neq 1$. Let $f(x) = \frac{1}{x(\ln(x))^a}$. This function is continuous, positive and decreasing for $x \geq 4$, so the Integral Test applies. Now,

$$\begin{aligned}\int_4^\infty \frac{dx}{x(\ln(x))^a} &= \lim_{R\to\infty} \int_4^R \frac{dx}{x(\ln(x))^a} \\ &= \lim_{R\to\infty} \int_4^R \frac{du}{u^a} \\ &= \frac{1}{1-a} \lim_{R\to\infty} \left(\frac{1}{(\ln(R))^{a-1}} - \frac{1}{(\ln(4))^{a-1}}\right)\end{aligned}$$

Because

$$\lim_{R\to\infty} \frac{1}{(\ln(R))^{a-1}} = \begin{cases} \infty, & 0 < a < 1 \\ 0, & a > 1 \end{cases}$$

conclude that the integral diverges when $0 < a < 1$ and converges when $a > 1$. Therefore,

$$\sum_{n=4}^{\infty} \frac{1}{n(\ln(n))^a} \text{ converges for } a > 1 \text{ and diverges for } 0 < a < 1$$

Next, consider the case $a = 1$. The series becomes $\sum_{n=4}^{\infty} \frac{1}{n\ln(n)}$. Let $f(x) = \frac{1}{x\ln(x)}$. For $x \geq 4$, this function is continuous positive, and decreasing, so the Integral Test applies. Use ing the substitution $u - \ln(x)$, $du = \frac{1}{x}\,dx$, we find

$$\begin{aligned}\int_4^\infty \frac{dx}{x\ln(x)} &= \lim_{R\to\infty} \int_4^R \frac{dx}{x\ln(x)} \\ &= \lim_{R\to\infty} \int_4^R \frac{du}{u} \\ &= \lim_{R\to\infty} \left(\ln(\ln(R)) - \ln(\ln(4))\right) \\ &= \infty\end{aligned}$$

The integral diverges; hence, the series also diverges.

To summarize:

$$\sum_{n=4}^{\infty} \frac{1}{n(\ln(n))^a} \text{ converges for } a > 1 \text{ and diverges for } 0 < a \leq 1$$

(EX) 10.3.07 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | INCORRECT FEEDBACK | DEFAULT FEEDBACK

Use the Integral Test to determine whether the infinite series $\sum_{n=1}^{\infty} \frac{11}{n^2+1}$ is convergent.

(Use symbolic notation and fractions where needed.)

$$\int_1^\infty \frac{11}{x^2+1}\,dx = \frac{11\pi}{4}$$

Solution

Integral Test

Let $a_n = f(n)$, where f is a positive, decreasing, and continuous function of x for $x \geq 1$.

- If $\int_1^\infty f(x)\,dx$ converges, then $\sum_1^\infty a_n$ converges.
- If $\int_1^\infty f(x)\,dx$ diverges, then $\sum_1^\infty a_n$ diverges.

Let $f(x) = \frac{11}{x^2+1}$, then the Integral Test applies because f is positive, decreasing, and continuous for $x \geq 1$. So,

$$\int_1^\infty \frac{11}{x^2+1}\,dx = \lim_{R\to\infty} \int_1^R \frac{11}{x^2+1}\,dx = 11\lim_{R\to\infty} \left(\tan^{-1}(R) - \frac{\pi}{4}\right) = 11\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = \frac{11\pi}{4}$$

The integral converges. Therefore, the series $\sum_{n=1}^{\infty} \frac{11}{n^2+1}$ converges.

(EX) 10.3.50 (Rogawski 4e ET)

QUESTION VIEW | SOLUTION | INCORRECT FEEDBACK | DEFAULT FEEDBACK

Let $a_n = \frac{5n - \cos(n)}{n^3}$ and $b_n = \frac{1}{n^2}$. Calculate the following limit.

(Express numbers in exact form. Use symbolic notation and fractions where nee

$L = \lim_{n\to\infty} \frac{a_n}{b_n} =$ 5

Solution

Apply the Limit Comparison Test with $a_n = \frac{5n - \cos(n)}{n^3}$ and $b_n = \frac{1}{n^2}$:

$$L = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{5n-\cos(n)}{n^3}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{5n - \cos(n)}{n} = \lim_{n\to\infty} \left(5 - \frac{\cos(n)}{n}\right) = 5$$

The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a convergent p-series. Because L exists, by the Limit Comparison Test we can conclude that the series $\sum_{n=1}^{\infty} \frac{5n - \cos(n)}{n^3}$ also converges.

(EX) 10.3.37 (Rogawski 4e ET) (ESF)

QUESTION VIEW | SOLUTION | INCORRECT FEEDBACK - 1 | DEFAULT FEEDBACK

Let $a_n = \frac{n^2}{n^4 - 4}$ and $b_n = \frac{1}{n^2}$.

Calculate the following limit.

$$\lim_{n\to\infty} \frac{a_n}{b_n}$$

(Give an exact answer. Use symbolic notation and fractions where needed. Enter

Solution

Calculate the limit of $\frac{a_n}{b_n}$ as n approaches ∞.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n^2}{n^4-4}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^4}{n^4 - 4} = \lim_{n\to\infty} \frac{1}{1 - \frac{4}{n^4}}$$

Therefore, the limit $\lim_{n\to\infty} \frac{a_n}{b_n}$ equals 1.

The series $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n^2}$ is a p-series with $p = 2 > 1$, so it converges.

Because the limit $\lim_{n\to\infty} \frac{a_n}{b_n}$ exists, by the Limit Comparison Test, the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{n^2}{n^4 - 4}$ also converges.