(EX) 10.3.50 (Rogawski 4e ET)

SOLUTION INCORRECT FEEDBACK DEFAULT FEEDBACK

Let $a_n = \frac{5n - \cos(n)}{n^3}$ and $b_n = \frac{1}{n^2}$. Calculate the following limit.

(Express numbers in exact form. Use symbolic notation and fractions where needed. Enter DNE if t

$L = \lim_{n\to\infty} \frac{a_n}{b_n} =$ 5

Apply the Limit Comparison Test with $a_n = \frac{5n - \cos(n)}{n^3}$ and $b_n = \frac{1}{n^2}$:

$$\begin{aligned} L &= \lim_{n\to\infty} \frac{a_n}{b_n} \\ &= \lim_{n\to\infty} \frac{\frac{5n-\cos(n)}{n^3}}{\frac{1}{n^2}} \\ &= \lim_{n\to\infty} \frac{5n - \cos(n)}{n} \\ &= \lim_{n\to\infty} \left(5 - \frac{\cos(n)}{n}\right) \\ &= 5 \end{aligned}$$

The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a convergent p-series. Because L exists, by the Limit Comparison Test we can conclude that the series $\sum_{n=1}^{\infty} \frac{5n - \cos(n)}{n^3}$ also converges.

(EX) 10.3.37 (Rogawski 4e ET) (ESF)

SOLUTION INCORRECT FEEDBACK

Let $a_n = \frac{n^2}{n^4 - 3}$ and $b_n = \frac{1}{n^2}$.

Calculate the following limit.

$$\lim_{n\to\infty} \frac{a_n}{b_n}$$

(Give an exact answer. Use symbolic

Calculate the limit of $\frac{a_n}{b_n}$ as n approaches ∞.

$$\begin{aligned} \lim_{n\to\infty} \frac{a_n}{b_n} &= \lim_{n\to\infty} \frac{\frac{n^2}{n^4-3}}{\frac{1}{n^2}} \\ &= \lim_{n\to\infty} \frac{n^4}{n^4 - 3} \\ &= \lim_{n\to\infty} \frac{1}{1 - \frac{3}{n^4}} \\ &= 1 \end{aligned}$$

Therefore, the limit $\lim_{n\to\infty} \frac{a_n}{b_n}$ equals 1.

The series $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n^2}$ is a p-series with $p = 2 > 1$, so it converges.

Because the limit $\lim_{n\to\infty} \frac{a_n}{b_n}$ exists, by the Limit Comparison Test, the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{n^2}{n^4 - 3}$ also converges.

(EX) 10.3.75 (Rogawski 4e ET)

SOLUTION INCORRECT FEEDBACK DEF

Let

$$a_n = \frac{5n^2 + 14n}{3n^4 - 5n^2 - 23}, \quad b_n = \frac{5}{3n^2}$$

Calculate the limit.

(Give an exact answer. Use symbolic notation

Substitute $a_n = \frac{5n^2 + 14n}{3n^4 - 5n^2 - 23}$ and $b_n = \frac{5}{3n^2}$ into the ratio and evaluate the limit.

$$\begin{aligned} \lim_{n\to\infty} \frac{a_n}{b_n} &= \lim_{n\to\infty} \left(\frac{\frac{5n^2+14n}{3n^4-5n^2-23}}{\frac{5}{3n^2}} \right) \\ &= \lim_{n\to\infty} \left(\frac{5n^2 + 14n}{3n^4 - 5n^2 - 23} \cdot \frac{3n^2}{5} \right) \\ &= \lim_{n\to\infty} \frac{15n^4 + 42n^3}{15n^4 - 25n^2 - 115} \\ &= \lim_{n\to\infty} \frac{15 + \frac{42}{n}}{15 - \frac{25}{n^2} - \frac{115}{n^4}} \\ &= \frac{15 + 0}{15 - 0 - 0} \\ &= 1 \end{aligned}$$

Therefore, $\lim_{n\to\infty} \frac{a_n}{b_n} = 1$.

Consider the infinite series.

$$\sum_{n=1}^{\infty} \frac{5}{3n^2} = \frac{5}{3} \cdot \sum_{n=1}^{\infty} \frac{1}{n^2}$$

The infinite series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges otherwise. Thuse, $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, so the series $\sum_{n=1}^{\infty} \frac{5}{3n^2}$ converges as well.

Because the sequences $\{a_n\}$ and $\{b_n\}$ are positive, the Limit Comparison Test can be applied. The Limit Comparison Test

Therefore, the series $\sum_{n=1}^{\infty} \frac{5n^2 + 14n}{3n^4 - 5n^2 - 23}$ converges because $\lim_{n\to\infty} \frac{a_n}{b_n} > 0$ and the series $\sum_{n=1}^{\infty} \frac{5}{3n^2}$ converges.

(EX) 10.3.43 (Rogawski 4e ET) (Video Feedback)

W SOLUTION INCORRECT FEEDBACK DEFAULT FEEDBACK

Let $a_n = \frac{3}{\sqrt{n} + \ln(n)}$ and $b_n = \frac{1}{\sqrt{n}}$. Calculate the following limit.

(Give an exact answer. Use symbolic notation and fractions where ne

$\lim_{n\to\infty} \frac{a_n}{b_n} =$ 3

Substitute $a_n = \frac{3}{\sqrt{n} + \ln(n)}$ and $b_n = \frac{1}{\sqrt{n}}$ into the ratio and evaluate the limit using L'Hôpital's Rule.

$$\begin{aligned} L &= \lim_{n\to\infty} \frac{a_n}{b_n} \\ &= \lim_{n\to\infty} \left(\frac{3}{\sqrt{n} + \ln(n)} \cdot \frac{\sqrt{n}}{1} \right) \\ &= \lim_{n\to\infty} \frac{3\sqrt{n}}{\sqrt{n} + \ln(n)} \\ &= \lim_{n\to\infty} \frac{(3\sqrt{n})'}{(\sqrt{n} + \ln(n))'} \\ &= \lim_{n\to\infty} \frac{\frac{3}{2\sqrt{n}}}{\frac{1}{2\sqrt{n}} + \frac{1}{n}} \\ &= \lim_{n\to\infty} \frac{3}{1 + \frac{2}{\sqrt{n}}} \\ &= \frac{3}{1 + 0} \\ &= 3 \end{aligned}$$

Therefore, $\lim_{n\to\infty} \frac{a_n}{b_n} = 3$.

Consider the infinite series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$. The infinite series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges otherwise.

Therefore, $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges.

Because the sequences $\{a_n\}$ and $\{b_n\}$ are positive, the Limit Comparison Test can be applied. Therefore, as L exists and $L > 0$, the series $\sum_{n=1}^{\infty} \frac{3}{\sqrt{n} + \ln(n)}$ also diverges.

Determine convergence or divergence of $\sum_{n=1}^{\infty} \frac{8^n}{9^n - 2n}$ using any method covered so far.

Limit Comparison Test

Let $\{a_n\}$ and $\{b_n\}$ be positive sequences. Assume that the following limit exists.

$$L = \lim_{n\to\infty} \frac{a_n}{b_n}$$

- If $L > 0$, then $\sum a_n$ converges if and only if $\sum b_n$ converges.
- If $L = \infty$ and $\sum a_n$ converges, then $\sum b_n$ converges.
- If $L = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.

Let

$$a_n = \frac{8^n}{9^n - 2n}$$
$$b_n = \frac{8^n}{9^n}$$

For large n, $a_n \approx b_n$. Thus,

$$L = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{8^n}{9^n-2n}}{\frac{8^n}{9^n}} = \lim_{n\to\infty} \frac{1}{1 - \frac{2n}{9^n}}$$

Now,

$$\lim_{n\to\infty} \frac{2n}{9^n} = \lim_{x\to\infty} \frac{2x}{9^x} = \lim_{x\to\infty} \frac{2}{9^x \ln(9)} = 0$$

Therefore,

$$L = \lim_{n\to\infty} \frac{a_n}{b_n} = \frac{1}{1 - 0} = 1$$

The series $\sum_{n=1}^{\infty} \left(\frac{8}{9}\right)^n$ is a convergent geometric series. Because L exists, by the Limit Comparison Test conclude that the series $\sum_{n=1}^{\infty} \frac{8^n}{9^n - 2n}$ also converges.

Determine convergence or divergence of $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^{23/24}}\right)$ using any method.

The series converges

Apply the limit Comparison Test with

$$a_n = \sin\left(\frac{1}{n^{23/24}}\right)$$
$$b_n = \frac{1}{n^{23/24}}$$

So,

$$L = \lim_{n\to\infty} \frac{\sin\left(\frac{1}{n^{23/24}}\right)}{\frac{1}{n^{23/24}}} = \lim_{u\to 0} \frac{\sin(u)}{u} = 1$$

The p-series diverges. Because $L = 1$, by the Limit Comparison Test, conclude that the series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^{23/24}}\right)$ also diverges.

(EX) 10.3.41 (Rogawski 4e ET)

SOLUTION - 1 INCORRECT FEEDBACK - 1 DE

Use the Limit Comparison Test for

$$\sum_{n=23}^{\infty} a_n = \sum_{n=23}^{\infty} \frac{6n^2 + 12}{n(n-19)(n-1)}$$

to prove convergence or divergence of the infinite series.

For large n,

$$a_n = \frac{6n^2 + 12}{n(n-19)(n-1)} \approx \frac{6n^2}{n^3} = \frac{6}{n}$$

so apply the Limit Comparison Test with

$$b_n = \frac{1}{n}$$

Now find

$$L = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{6n^2+12}{n(n-19)(n-1)}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{6n^3 + 12n}{n(n-19)(n-1)} = 6$$

The series $\sum_{n=1}^{\infty} \frac{1}{n}$ is a p-series with $p = 1$, so it diverges, hence, the series $\sum_{n=23}^{\infty} \frac{1}{n}$ also diverges. Because $L > 0$ exists, by the Limit Comparison Test conclude that the series $\sum_{n=23}^{\infty} \frac{6n^2+12}{n(n-19)(n-1)}$ diverges.

Use the Direct Comparison Test to determine, which of the statements is true for the infinite series $\sum_{n=1}^{\infty} \frac{(\sin(n))^8}{n^{16}}$.

The series diverges.

First, for $n \geq 1$,

$$0 \leq \frac{(\sin(n))^8}{n^{16}} \leq \frac{1}{n^{16}}$$

The larger series $\sum_{n=1}^{\infty} \frac{1}{n^{16}}$ converges because it is a p-series with $p = 16 > 1$. By the Direct Comparison Test, the smaller series $\sum_{n=1}^{\infty} \frac{(\sin(n))^8}{n^{16}}$ also converges.

(EX) 10.3.51 (Rogawski 4e ET)

SOLUTION - 1 INCORRECT FEEDBACK - 1 DEFA

Write an inequlity comparing $\frac{n^2 - 5}{n^5 + 5}$ with $\frac{1}{n^3}$ for $n \geq 1$.

For any $n \geq 1$, the following inequality holds.

$$\frac{n^2 - 5}{n^5 + 5} < \frac{n^2}{n^5} = \frac{1}{n^3}$$

Consider the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^3}$. The infinite series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges otherwise. Thus, $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges.

By the Direct Comparison Test, if $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges, then $\sum_{n=1}^{\infty} \frac{n^2 - 5}{n^5 + 5}$ also converges.

(EX) 10.3.20 (Rogawski 4e ET) (Video Feedback)

N SOLUTION - 1 INCORRECT FEEDBACK - 1 DEFAULT FEEDB

Consider the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + 5n - 1}}$ and $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$.

Write an inequality comparing $\frac{1}{\sqrt{n^3 + 5n - 1}}$ to $\frac{1}{n^{3/2}}$ for $n \geq 1$.

Let $f(n) = \frac{1}{\sqrt{n^3+5n-1}}$ and $g(n) = \frac{1}{n^{3/2}}$. Because $5n - 1 > 0$ for any $n \geq 1$ and the numerators of the fractions are the same, the following inequality holds.

$$\frac{1}{n^{3/2}} = \frac{1}{\sqrt{n^3}} > \frac{1}{\sqrt{n^3 + 5n - 1}}$$

Consider the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$. The infinite series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges otherwise. Therefore, using $p = \frac{3}{2}$, the series $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ converges.

By the Comparison Test, $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + 5n - 1}}$ also converges.

HW19 10.4

12 questions

Course Info | Instructor Name | Student Name

Question 1 of 12

Consider the following series

$$\frac{1}{2} - \frac{1}{3} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{2^3} - \frac{1}{3^3} + \cdots$$

Can the Alternating Series Test be applied? Why or why not?

- ○ No, the sequence is not decreasing.
- ○ Yes, the series is alternating and the values in each term are getting smaller.
- ☒ No, every other term is not defined similarly to the previous term.
- ○ Yes, the series appears to be converging.

Find the general term $\{a_n\}$ for the series.

(Enter your answer in terms of n so that n starts at 1.)

$\left\{ \frac{1}{2^n} - \frac{1}{3^n} \right\}$

$a_n =$

Find the sum of the series if it exists.

(Enter an exact answer. Use symbolic notation or fractions where needed. Enter DNE if the sum does not exist.)

both geometric

$$\sum \frac{1}{2^n} - \frac{1}{3^n} = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n - \left(\frac{1}{3}\right)^n = \frac{\frac{1}{2}}{1-\frac{1}{2}} - \frac{\frac{1}{3}}{1-\frac{1}{3}} = 1 - \frac{1}{3} \cdot \frac{3}{2} = \frac{1}{2}$$

$\sum_{n=1}^{\infty} a_n =$

Determine convergence or divergence by any method.

$$\sum_{n=0}^{\infty} \frac{(-1)^n n}{\sqrt{n^2+5}}$$

$$\lim_{n\to\infty} \frac{n}{\sqrt{n^2+5}} \quad 1 \quad \rightarrow$$

The series

- ○ converges, since the terms alternate.
- ○ converges, since the terms are smaller than $\frac{1}{n}$.
- ☒ diverges, since $\lim_{n\to\infty} a_n \neq 0$.
- ○ converges, since $\lim_{n\to\infty} a_n = 0$.
- ○ diverges, since the terms are larger than $\frac{1}{n^2}$.

Question 3 of 12

Determine convergence or divergence by any method.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n^{1/2}(\ln(n))^4}$$

The series

- [x] converges. AST
- [ ] diverges.

Question 4 of 12

Determine convergence or divergence by any method.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^3+1}}$$

The series

- [x] converges. AST
- [ ] diverges.

Consider the series.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln\left(n^3\right)} = \frac{(-1)^n}{3n \cdot \ln(n)}$$

Determine whether the series converges absolutely, conditionally, or not at all.

- [x] The series converges conditionally. (integral test)
- [ ] The series does not converge.
- [ ] The series converges absolutely.

Consider the series.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{1/3}}$$

Determine whether the series converges absolutely, conditionally, or not at all.

- ○ The series converges absolutely.
- ○ The series does not converge.
- ○ The series converges conditionally.

P

Determine convergence or divergence of $\sum_{n=5}^{\infty} \left(\frac{3}{4}\right)^{-n}$ using any method covered so far.

- ○ The series converges.
- ⊗ The series diverges.

$= \left(\frac{4}{3}\right)^n$

div. test

Consider the series.

$$\sum_{n=4}^{\infty} \ln\left(1 + \frac{1}{n}\right) = \ln\left(\frac{n+1}{n}\right) = \ln(n+1) - \ln(n)$$

Find a simplified expression for the partial sum S_k.

(Express numbers in exact form. Use symbolic notation and fractions where needed.)

$\ln(5) - \ln(4) + \ln(6) - \ln(5) + \ln(7) - \ln(6) + \ldots$

$\ln(k+1) - \ln(4) \to \infty$

$S_k =$

Determine the convergence or divergence of the series.

- It is not possible to determine the convergence or divergence of the series since $\lim_{k\to\infty} S_k$ does not exist.
- The series converges since $\lim_{k\to\infty} S_k$ is finite.
- The series diverges since $\lim_{k\to\infty} S_k$ is infinite.

Determine convergence or divergence of $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^{6/7}}\right)$ using any method.

- ◯ The series diverges.
- ◯ The series converges.

L.C.T. $\lim_{n\to\infty} \frac{\sin\left(\frac{1}{n^{6/7}}\right)}{\frac{1}{n^{6/7}}} = \lim_{u\to 0} \frac{\sin u}{u} = 1$

$\Rightarrow$ since $\sum \frac{1}{n^{6/7}}$ div., series diverges

Find the limit $\lim\limits_{k\to\infty} 4^{2/k}$.

$\lim\limits_{k\to\infty} 4^{2/k} =$ 1

Deteremine whether $\sum_{k=1}^{\infty} 4^{2/k}$ converges or diverges.

- ○ The series diverges because $\lim\limits_{k\to\infty} 4^{2/k}$ is infinite.
- ○ The series converges because $\lim\limits_{k\to\infty} 4^{2/k}$ is finite.
- ☒ The series diverges because $\lim\limits_{k\to\infty} 4^{2/k}$ is nonzero.

Let $a_n = \dfrac{1}{5^n - 3^n}$ and $b_n = \dfrac{1}{5^n}$. Calculate the limit.

$$\lim_{n\to\infty} \frac{a_n}{b_n}$$

(Give an exact answer. Use symbolic notation and fractions where needed. Enter DNE if the limit does not exist.)

$$y = \frac{1}{5^n - 3^n} \cdot \frac{5^n}{1} = \frac{5^n}{5^n - 3^n} \begin{matrix} \div 5^n \\ \div 5^n \end{matrix} = \frac{1}{1 - \frac{3^n}{5^n}} = \frac{1}{1 - \left(\frac{3}{5}\right)^n} \to 1 \text{ as } n \to \infty$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \boxed{}$$

$\Sigma \frac{1}{5^n} = \Sigma \left(\frac{1}{5}\right)^n \to$ geometric converge. $\Rightarrow \therefore$ Converge L.C.T.

Use the limit to identify whether the series $\sum_{n=1}^{\infty} \dfrac{3}{5^n - 3^n}$ converges or diverges.

- ○ $\sum_{n=1}^{\infty} \dfrac{3}{5^n - 3^n}$ diverges because $\lim_{n\to\infty} \dfrac{a_n}{b_n}$ is infinite and $\sum_{n=1}^{\infty} b_n$ diverges.
- ○ $\sum_{n=1}^{\infty} \dfrac{3}{5^n - 3^n}$ converges because $\lim_{n\to\infty} \dfrac{a_n}{b_n}$ is finite and $\sum_{n=1}^{\infty} b_n$ diverges.
- ○ $\sum_{n=1}^{\infty} \dfrac{3}{5^n - 3^n}$ converges because $\lim_{n\to\infty} \dfrac{a_n}{b_n}$ is finite and $\sum_{n=1}^{\infty} b_n$ converges.
- ○ $\sum_{n=1}^{\infty} \dfrac{3}{5^n - 3^n}$ diverges because $\lim_{n\to\infty} \dfrac{a_n}{b_n}$ is finite and $\sum_{n=1}^{\infty} b_n$ diverges.

Write an inequality relating $\frac{e^{-n}}{n^2}$ to $\frac{1}{n^2}$ for $n \geq 1$.

(Express numbers in exact form. Use symbolic notation and fractions where needed.)

$$\frac{e^{-n}}{n^2} = \frac{1}{e^n \cdot n^2} < \frac{1}{n^2}$$

inequality:

Use the above inequality to determine if the series $\sum_{n=1}^{\infty} \frac{(-1)^n e^{-n}}{4n^2}$ converges or diverges.

- The series diverges.
- The series converges conditionally.
- The series converges absolutely. DCT

Question 1 of 16

Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n}{3n+7}$$

Consider the shown justification.

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{\frac{n+1}{3n+10}}{\frac{n}{3n+7}} \right|$$

$$= \lim_{n\to\infty} \frac{n+1}{3n+10} \cdot \frac{3n+7}{n}$$

$$= \lim_{n\to\infty} \frac{3n^2+10n+7}{3n^2+10n}$$

$$= 1$$

Thus, the series $\sum_{n=1}^{\infty} \frac{n}{3n+7}$ diverges.

Determine if the shown justification is valid.

- ◯ This justification is valid.
- ◯ This justification is not valid, because the Root Test is inconclusive when the limit tends to 1.
- ◯ This justification is not valid, because the series converges to 1.
- ☒ This justification is not valid, because the Ratio Test is inconclusive when the limit tends to 1.
- ◯ This justification is not valid, because the Root Test should have been applied instead.

Calculate the limit.

$$\lim_{n\to\infty} \frac{n}{3n+7}$$

(Express numbers in exact form. Use symbolic notation and fractions where needed.)

$\lim_{n\to\infty} \frac{n}{3n+7} =$ [ ]

Use the limit to complete the statement whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n}{3n+7}$$

The series [ ] by the [ ].

Question 2 of 16

Find $\sqrt[k]{|a_k|}$ for the series.

$$\sum_{k=0}^{\infty} \left(\frac{k}{k+15}\right)^k$$

(Express numbers in exact form. Use symbolic notation and fractions where needed.)

$\sqrt[k]{|a_k|} =$ [ ]

Use the Root Test to determine the correct stetement.

- The Root Test is inconclusive.
- The series diverges.
- The series converges.

Question 3 of 16

Given the series

$$\sum_{n=1}^{\infty} \frac{(n!)^3}{(8n)!}$$

$$\frac{(n+1)!^3}{(8(n+1))!} \cdot \frac{(8n)!}{(n!)^3} = \frac{(n+1)^3 \cancel{n!^3}\, \cancel{(8n)!}}{(8n+8)(8n+7)\cdots(8n+1)\cancel{(8n)!}\, \cancel{n!^3}} \approx \frac{n^3}{n^8} \to 0$$

find the ratio $\left|\frac{a_{n+1}}{a_n}\right|$.

(Express numbers in exact form. Use symbolic notation and fractions where needed.)

$\left|\frac{a_{n+1}}{a_n}\right| =$

Use the Ratio Test to determine the correct statement.

- The series diverges.
- The Ratio Test is inconclusive.
- The series converges.

Question 4 of 16

Given the series

$$\sum_{n=1}^{\infty} \frac{7^n}{n}$$

$$\frac{\overset{= 7^n \cdot 7}{7^{n+1}}}{n+1} \cdot \frac{n}{7^n} = \frac{7n}{n+1} \rightarrow 7$$

find the ratio $\left|\frac{a_{n+1}}{a_n}\right|$.

(Express numbers in exact form. Use symbolic notation and fractions where needed.)

$\left|\frac{a_{n+1}}{a_n}\right| =$

Use the Ratio Test to determine the correct statement.

- The series converges.
- The series diverges.
- The Ratio Test is inconclusive.

Question 5 of 16

Use the Root Test to determine convergence or divergence (or state that the test is inconclusive) of the given series.

$$\sum_{n=0}^{\infty} \frac{1}{6^n}$$

Choose the correct answer.

- ○ The series converges conditionally.
- ○ The series diverges.
- ☒ The series converges absolutely.
- ○ The test is inconclusive.

Question 6 of 16

Apply the Ratio Test to determine convergence or divergence of the given series, or state that the Ratio Test is inconclusive.

$$\sum_{n=1}^{\infty} \frac{5n+2}{7n^3+1}$$

use L.C.T.

$$\frac{5n+2}{7n^3+1} \cdot \frac{7n^3}{5n} = 1$$

conv

- The series converges absolutely.
- The series diverges.
- The test is inconclusive.
- The series converges but does not converge absolutely.

Question 7 of 16

Determine convergence or divergence of the series using any method.

$$\sum_{n=1}^{\infty} \frac{n^3}{n!} \qquad \frac{(n+1)^3}{(n+1)!} \cdot \frac{n!}{n^3} = \frac{(n+1)^3}{(n+1)\,n^3} \approx \frac{1}{n} \longrightarrow 0$$

$\Rightarrow$ converge

Choose the correct answer.

- ○ The convergence of the series cannot be determined.
- ○ The series diverges.
- ○ The series converges conditionally.
- ⊗ The series converges absolutely.

Determine convergence or divergence of the series using any method.

$$\sum_{n=1}^{\infty} \frac{n^{11}}{n!}$$

Choose the correct answer.

- ○ The convergence of the series cannot be determined.
- ○ The series diverges.
- ○ The series converges conditionally.
- ○ The series converges absolutely.

Question 9 of 16

Use the Root Test to determine convergence or divergence (or state that the test is inconclusive) of the given series.

$$\sum_{k=0}^{\infty}\left(\frac{k}{3k+1}\right)^k$$

Choose the correct answer.

- The series diverges.
- The series converges absolutely.
- The test is inconclusive.
- The series converges conditionally.

With $a_k = \left(\frac{k}{2k+1}\right)^k$,

$$\sqrt[k]{a_k} = \sqrt[k]{\left(\frac{k}{2k+1}\right)^k} = \frac{k}{2k+1}$$

Consider the limit of the kth roots at infinity.

$$L = \lim_{k\to\infty} \sqrt[k]{|a_k|} = \lim_{k\to\infty}\left(\frac{k}{2k+1}\right) = \frac{1}{2} < 1$$

Therefore, the series $\sum_{k=0}^{\infty}\left(\frac{k}{2k+1}\right)^k$ converges absolutely by the Root Test.

Question 10 of 16

Assume that $\left|\frac{a_{n+1}}{a_n}\right|$ converges to $\rho = \frac{1}{6}$. What can you say about the convergence of the given series?

$$\sum_{n=1}^{\infty} a_n^2$$

Choose the correct answer.

- The series converges conditionally.
- The test is inconclusive.
- The series diverges.
- The series converges absolutely.

Use the Ratio Test. Let $b_n = a_n^2$. Then,

$$\rho_b = \lim_{n\to\infty}\left|\frac{b_{n+1}}{b_n}\right| = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|^2 = \left(\frac{1}{6}\right)^2 = \frac{1}{36}$$

Since $\rho_b < 1$, the series $\sum_{n=1}^{\infty} a_n^2$ converges absolutely by the Ratio Test.

Question 11 of 16

Assume that $\left|\frac{a_{n+1}}{a_n}\right|$ converges to $\rho = \frac{1}{4}$. What can you say about the convergence of the given series?

$$\sum_{n=1}^{\infty} 5^n a_n$$

set $b_n = 5^n a_n$

then $b_{n+1} = 5^{n+1} a_{n+1}$

$\frac{b_{n+1}}{b_n} = \frac{5^{n+1} a_{n+1}}{5^n a_n} = 5\left|\frac{a_{n+1}}{a_n}\right| = \frac{5}{4} > 1$ div.

Choose the correct answer.

- ○ The series converges conditionally.
- ☒ The series diverges.
- ○ The test is inconclusive.
- ○ The series converges absolutely.

Apply the Ratio Test to determine convergence or divergence of the given series, or state that the Ratio Test is inconclusive.

$$\sum_{n=1}^{\infty} \frac{7^n}{n!}$$

$\frac{7^{n+1}}{(n+1)!} \cdot \frac{n!}{7^n} = \frac{7}{n+1} \to 0 < 1 \Rightarrow$ converge

- The series converges absolutely.
- The test is inconclusive.
- The series converges but does not converge absolutely.
- The series diverges.

Question 13 of 16

The following limit could be helpful to answer the question.

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$

$\left(\frac{n+1}{n}\right)^n$ " $\frac{(n+1)^n}{n^n}$

$$\frac{(2(n+1))!}{(n+1)^{n+1}} \cdot \frac{n^n}{(2n)!} = \frac{(2n+2)(2n+1)n^n}{(n+1)^{n+1}}$$

$$= \frac{(2n+2)(2n+1)n^n}{(n+1)^n\cdot(n+1)} \div \frac{n^n}{n^n}$$

Does $\sum_{n=1}^{\infty}\frac{(2n)!}{n^n}$ converge or diverge?

$$= \lim_{n\to\infty}\frac{(2n+2)(2n+1)}{\frac{(n+1)^n}{n^n}\cdot(n+1)} = \lim_{n\to\infty}\frac{\overset{2(n+1)}{(2n+2)}(2n+1)}{e\cdot(n+1)}$$

$$\approx \lim_{n\to\infty}\frac{2(2n+1)}{e} \to \infty$$

- ~~The series converges absolutely.~~
- The test is inconclusive.
- The series converges conditionally.
- The series diverges.

Question 14 of 16

The following limit could be helpful to answer the question.

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$

$$\frac{(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{n!} = \frac{(n+1)\,n^n}{(n+1)(n+1)^n} = \left(\frac{n}{n+1}\right)^n = \left(1+\frac{1}{n}\right)^n \to e$$

Does $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ converge or diverge?

- [x] The series diverges.
- [ ] The series converges absolutely.
- [ ] The test is inconclusive.
- [ ] The series converges conditionally.

Question 15 of 16

Use the Root Test to determine convergence or divergence (or state that the test is inconclusive) of the given series.

$$\sum_{n=3}^{\infty}\left(1+\frac{1}{n}\right)^{-n^2} = \sum_{3}^{\infty}\Big[\underbrace{\left(1+\frac{1}{n}\right)^{-n}}_{a_n}\Big]^{n}$$

root $\sqrt[n]{a_n}$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{-n} = \left[\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}\right]^{-1} = e^{-1} = \frac{1}{e} < 0$$

Choose the correct answer.

- ○ The test is inconclusive.
- ○ The series diverges.
- ☒ The series converges absolutely.
- ○ The series converges conditionally.

Question 16 of 16

Solve for the value of k that makes the series converge.

$$\sum_{n=1}^{\infty} \frac{7^n}{n^k} \qquad \frac{7^{n+1}}{(n+1)^k} \cdot \frac{n^k}{7^n} = \frac{7n^k}{(n+1)^k} \to 7$$

(Use symbolic notation and fractions where needed. If such value does not exist, enter DNE.)

$k =$ DNE