Substitute $a_n = \frac{5n^2 + 14n}{3n^4 - 5n^2 - 23}$ and $b_n = \frac{5}{3n^2}$ into the ratio and evaluate the limit. (EX) 10.3.75 (Rogawski 4e ET)

SOLUTION INCORRECT FEEDBACK DEF,
$$\lim_{n\to\infty}\frac{a_n}{b_n}=\lim_{n\to\infty}\left(\frac{\frac{5n^2+14n}{3n^2-23}}{\frac{3n^2}{3n^2}}\right) = \frac{1}{15-0-0}$$
Let
$$a_n=\frac{5n^2+14n}{3n^4-5n^2-23},\quad b_n=\frac{5}{3n^2} = \lim_{n\to\infty}\left(\frac{5n^2+14n}{3n^4-5n^2-23},\frac{3n^2}{5n^2}\right) = 1$$

$$\lim_{n\to\infty}\frac{a_n}{b_n}=\lim_{n\to\infty}\left(\frac{5n^2+14n}{3n^4-5n^2-23},\frac{3n^2}{5}\right) = 1$$
Consider the infinite series $\frac{a_n}{b_n}=1$.
Calculate the limit.

> (EX) 10.3.43 (Rogawski 4e ET) (Video Feedback) Substitute $a_n=\frac{3}{\sqrt{n}+\ln{(n)}}$ and $b_n=\frac{1}{\sqrt{n}}$ into the ratio and evaluate the limit using L'Hôpital's Rule SOLUTION $L = \lim_{n \to \infty} \frac{a_n}{h}$ Let $a_n = \frac{3}{\sqrt{n} + \ln{(n)}}$ and $b_n = \frac{1}{\sqrt{n}}$. Calculate the following limit. $= \lim_{n \to \infty} \left(\frac{3}{\sqrt{n} + \ln{(n)}} \cdot \frac{\sqrt{n}}{1} \right)$ $= \lim_{n \to \infty} \frac{\frac{3}{2\sqrt{n}}}{\frac{1}{2\sqrt{n}} + \frac{1}{n}}$ (Give an exact answer. Use symbolic notation and fractions where ne $= \lim_{n \to \infty} \frac{3}{1 + \frac{2}{\sqrt{n}}}$ Consider the infinite series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$. The infinite series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if p>1 and diverges otherwise.

Determine convergence or divergence of $\sum_{n=1}^{\infty} \frac{8^n}{9^n - 2n}$ using any method covered so far.

Limit Comparison Test

(Give an exact answer. Use symbolic

(Give an exact answer. Use symbolic notation :

Let

Let $\{a_n\}$ and $\{b_n\}$ be positive sequences. Assume that the following limit exists.

$$\{b_n\}$$
 be positive sequences. Assume that the following limit exist a_n

- If L > 0, then $\sum a_n$ converges if and only if $\sum b_n$ converges.
- If $L = \infty$ and $\sum a_n$ converges, then $\sum b_n$ converges.
- If L = 0 and $\sum b_n$ converges, then $\sum a_n$ converges.

$$a_n = \frac{8^n}{9^n - 2n}$$
$$b_n = \frac{8^n}{9^n}$$

For large n, $a_n \approx b_n$. Thus,

$$L = \lim_{n \to \infty} \frac{a_n}{b_n} = \lim_{n \to \infty} \frac{\frac{8^n}{9^n - 2n}}{\frac{8^n}{9^n}} = \lim_{n \to \infty} \frac{1}{1 - \frac{2n}{9^n}}$$

$$\lim_{n \to \infty} \frac{2n}{9^n} = \lim_{x \to \infty} \frac{2x}{9^x} = \lim_{x \to \infty} \frac{2}{9^x \ln(9)} = 0$$

Therefore,

$$L = \lim_{n \to \infty} \frac{a_n}{b_n} = \frac{1}{1 - 0} = 1$$

The series $\sum_{n=1}^{\infty} \frac{\left(\frac{8}{9}\right)^n}{\frac{8^n}{9^n-2n}}$ also converges. Because L exists, by the Limit Comparison Test conclude that the series $\sum_{n=1}^{\infty} \frac{8^n}{\frac{9^n-2n}{9^n-2n}}$ also converges.

Determine convergence or divergence of $\sum_{n=1}^{\infty} \sin(\frac{1}{n^{23/24}})$ using any method.

The series converges

Apply the limit Comparison Test with

$$a_n = \sin\left(\frac{1}{n^{23/24}}\right)$$
$$b_n = \frac{1}{n^{23/24}}$$

So,

$$L = \lim_{n \to \infty} \frac{\sin(\frac{1}{n^{23/24}})}{\frac{1}{n^{23/24}}} = \lim_{u \to 0} \frac{\sin(u)}{u} = 1$$

The *p*-series diverges. Because L=1, by the Limit Comparison Test, conclude that the series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^{30/2}}\right)$ also diverges.

(EX) 10.3.41 (Rogawski 4e ET)



SOLUTION - 1 +

INCORRECT FEEDBACK - 1 ▼ D

Use the Limit Comparison Test for

$$\sum_{n=23}^{\infty} a_n = \sum_{n=23}^{\infty} \frac{6n^2 + 12}{n(n-19)(n-1)}$$

to prove convergence or divergence of the infinite series.

For large n

$$a_n = \frac{6n^2 + 12}{n(n-19)(n-1)} \approx \frac{6n^2}{n^3} = \frac{6}{n}$$

so apply the Limit Comparison Test with

$$b_n = \frac{1}{r}$$

Now find

$$L = \lim_{n \to \infty} \frac{a_n}{b_n} = \lim_{n \to \infty} \frac{\frac{6n^2 + 12}{n(n - 19)(n - 1)}}{\frac{1}{n}} = \lim_{n \to \infty} \frac{6n^3 + 12n}{n(n - 19)(n - 1)} = 6$$

The series $\sum_{n=1}^{\infty} \frac{1}{n}$ is a p-series with p=1, so it diverges, hence, the series $\sum_{n=23}^{\infty} \frac{1}{n}$ also diverges. Because L>0 exists, by the Limit Comparison Test conclude that the series $\sum_{n=23}^{\infty} \frac{6n^2+12}{n(n-19)(n-1)}$ diverges.

Use the Direct Comparison Test to determine, which of the statements is true for the infinite series $\sum_{n=1}^{\infty} \frac{(\sin(n))^8}{n^{16}}$. The series diverges.

First, for $n \ge 1$,

$$0 \le \frac{(\sin(n))^8}{n^{16}} \le \frac{1}{n^{16}}$$

The larger series $\sum_{n=1}^{\infty} \frac{1}{n!^6}$ converges because it is a *p*-series with p=16>1. By the Direct Comparison Test, the smaller series $\sum_{n=1}^{\infty} \frac{(\sin(n))^8}{n!^6}$ also converges.

(EX) 10.3.51 (Rogawski 4e ET)



SOLUTION - 1 ▼

INCORRECT FEEDBACK - 1 ▼

 $n^2 - 5$ n^2 1

For any $n \ge 1$, the following inequality holds

$$\frac{n^2 - 5}{n^5 + 5} < \frac{n^2}{n^5} = \frac{1}{n^5}$$

Consider the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^3}$. The infinite series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if p > 1 and diverges otherwise. Thus,

$$\sum_{n=1}^{\infty} \frac{1}{n^3} \text{ converge}$$

By the Direct Comparison Test, if $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges, then $\sum_{n=1}^{\infty} \frac{n^2-5}{n^5+5}$ also converges.

(EX) 10.3.20 (Rogawski 4e ET) (Video Feedback)



V SOLUTION -

INCORRECT FEEDBACK - 1 ▼

DEFAULT FEEDB/

DEFA

Consider the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + 5n - 1}}$ and $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$.

Write an inequality comparing $\frac{n^2 - 5}{n^5 + 5}$ with $\frac{1}{n^3}$ for $n \ge 1$.

Write an inequality comparing $\frac{1}{\sqrt{n^3 + 5n - 1}}$ to $\frac{1}{n^{3/2}}$ for $n \ge 1$.

Let $f(n) = \frac{1}{\sqrt{n^2 + 5n - 1}}$ and $g(n) = \frac{1}{n^{3/2}}$. Because 5n - 1 > 0 for any $n \ge 1$ and the numerators of the fractions are the same, the following inequality holds.

$$\frac{1}{n^{3/2}} = \frac{1}{\sqrt{n^3}} > \frac{1}{\sqrt{n^3 + 5n - 1}}$$

Consider the infinite series $\sum_{n=1}^{\infty}\frac{1}{n^{2}}$. The infinite series $\sum_{n=1}^{\infty}\frac{1}{n^{p}}$ converges if p>1 and diverges otherwise. Therefore, using $p=\frac{3}{2}$, the series $\sum_{n=1}^{\infty}\frac{1}{n^{p}}$ converges.

By the Comparison Test, $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + 5n - 1}}$ also converges.

HW19 10.4

12 questions

Course Info

Instructor Name

Student Name

Question 1 of 12

Consider the following series

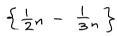
$$\frac{1}{2} - \frac{1}{3} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{2^3} - \frac{1}{3^3} + \cdots$$

Can the Alternating Series Test be applied? Why or why not?

- No, the sequence is not decreasing.
- Yes, the series is alternating and the values in each term are getting smaller.
- No, every other term is not defined similarly to the previous term.
- Yes, the series appears to be converging.

Find the general term $\{a_n\}$ for the series.

(Enter your answer in terms of *n* so that *n* starts at 1.) $\left\{\frac{1}{2}n - \frac{1}{3}n\right\}$





Find the sum of the series if it exists.

(Enter an exact answer. Use symbolic notation or fractions where needed. Enter DNE if the sum does not exist.)

both geometric
$$\frac{1}{2}$$

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n - \left(\frac{1}{3}\right)^n = \frac{\frac{1}{2}}{1 - \frac{1}{2}} - \frac{\frac{1}{3}}{1 - \frac{1}{3}} = 1 - \frac{1}{3} - \frac{3}{2} = \sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} a_n$$

Question 2 of 12

Determine convergence or divergence by any method.

$$\sum_{n=0}^{\infty} \frac{(-1)^n n}{\sqrt{n^2 + 5}}$$

The series

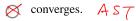
- onverges, since the terms alternate.
- O converges, since the terms are smaller than $\frac{1}{n}$.
- \bowtie diverges, since $\lim_{n\to\infty} a_n \neq 0$.
- O converges, since $\lim_{n\to\infty} a_n = 0$.
- O diverges, since the terms are larger than $\frac{1}{n^2}$.

Question 3 of 12

Determine convergence or divergence by any method.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n^{1/2} (\ln(n))^4}$$

The series



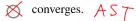
odiverges.

Question 4 of 12

Determine convergence or divergence by any method.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^3 + 1}}$$

The series



diverges.

Question 5 of 12

Consider the series.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln (n^3)} = \frac{(-1)^n}{3 n \ln (n)}$$

Determine whether the series converges absolutely, conditionally, or not at all.

The series converges conditionally. (integral text)

- The series does not converge.
- The series converges absolutely.

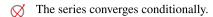
Question 6 of 12

Consider the series.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{1/3}}$$

Determine whether the series converges absolutely, conditionally, or not at all.

- The series converges absolutely.
- The series does not converge.





Question 7 of 12

Determine convergence or divergence of $\sum_{n=5}^{\infty} \left(\frac{3}{4}\right)^{-n}$ using any method covered so far.

The series converges.

The series diverges.

 $=\left(\frac{4}{3}\right)^n$

div. test

Question 8 of 12

Consider the series.

$$\sum_{n=4}^{\infty} \ln \left(1 + \frac{1}{n}\right) = \operatorname{ln}\left(\begin{array}{c} n+1 \\ \overline{n} \end{array}\right) = \operatorname{ln}(n+1) - \operatorname{ln}(n)$$

Find a simplified expression for the partial sum S_k .

(Express numbers in exact form. Use symbolic notation and fractions where needed.)

$$S_k =$$

Determine the convergence or divergence of the series.

- O It is not possible to determine the convergence or divergence of the series since $\lim_{k\to\infty} S_k$ does not exist.
- The series converges since $\lim_{k\to\infty} S_k$ is finite.
- The series diverges since $\lim_{k\to\infty} S_k$ is infinite.

Determine convergence or divergence of $\sum_{n=1}^{\infty} \sin(\frac{1}{n^{6/7}})$ using any method.

- The series diverges.
- The series converges.

verges.

L. C.T.
$$\frac{\sin(\pi \kappa_{17})}{\ln \pi} = \lim_{n \to \infty} \frac{\sin(\pi \kappa_{17$$

Question 10 of 12

Find the limit $\lim_{k\to\infty} 4^{2/k}$.

$$\lim_{k\to\infty}4^{2/k}=\boxed{1}$$

Determine whether $\sum_{k=1}^{\infty} 4^{2/k}$ converges or diverges.

- \bigcirc The series diverges because $\lim_{k\to\infty}4^{2/k}$ is infinite.
- $\bigcirc \quad \text{The series converges because } \lim_{k \to \infty} 4^{2/k} \text{ is finite.}$

Question 11 of 12

Let
$$a_n = \frac{1}{5^n - 3^n}$$
 and $b_n = \frac{1}{5^n}$. Calculate the limit.

$$\lim_{n\to\infty}\frac{a_n}{b_n}$$

(Give an exact answer. Use symbolic notation and fractions where needed. Enter DNE if the limit does not exist.)

$$y = \frac{1}{5n \cdot 3^n} \cdot \frac{5^n}{1} = \frac{5^n}{5^n - 3^n} \div \frac{5^n}{5^n} = \frac{1}{1 - (\frac{3}{5})^n} \longrightarrow 1$$
 as $n \to \infty$

$$\lim_{n\to\infty}\frac{a_n}{b_n}=$$

Use the limit to identify whether the series $\sum_{n=1}^{\infty} \frac{3}{5^n - 3^n}$ converges or diverges.

- $\sum_{n=1}^{\infty} \frac{3}{5^n 3^n} \text{ diverges because } \lim_{n \to \infty} \frac{a_n}{b_n} \text{ is infinite and } \sum_{n=1}^{\infty} b_n \text{ diverges.}$
- $\sum_{n=1}^{n=1} \frac{3}{5^n 3^n} \text{ converges because } \lim_{n \to \infty} \frac{a_n}{b_n} \text{ is finite and } \sum_{n=1}^{n=1} b_n \text{ diverges.}$ $\sum_{n=1}^{\infty} \frac{3}{5^n 3^n} \text{ converges because } \lim_{n \to \infty} \frac{a_n}{b_n} \text{ is finite and } \sum_{n=1}^{\infty} b_n \text{ converges.}$
- $\sum_{n=1}^{n-1} \frac{3}{5^n 3^n}$ diverges because $\lim_{n \to \infty} \frac{a_n}{b_n}$ is finite and $\sum_{n=1}^{\infty} b_n$ diverges.

Question 12 of 12

Write an inequality relating $\frac{e^{-n}}{n^2}$ to $\frac{1}{n^2}$ for $n \ge 1$.

(Express numbers in exact form. Use symbolic notation and fractions where needed.)

$$\frac{e^{-\eta}}{n^2} = \frac{1}{e^n \cdot n^2} < \frac{1}{n^2}$$

inequality:

Use the above inequality to determine if the series $\sum_{n=1}^{\infty} \frac{(-1)^n e^{-n}}{4n^2}$ converges or diverges.

- The series diverges.
- The series converges conditionally.
- \nearrow The series converges absolutely. $\mathbb{D} \subset \mathcal{T}$

Question 1 of 16

Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n}{3n+7}$$

Consider the shown justification.

$$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{\frac{n+1}{3n+10}}{\frac{n}{3n+7}} \right|$$

$$= \lim_{n \to \infty} \frac{n+1}{3n+10} \cdot \frac{3n+7}{n}$$

$$= \lim_{n \to \infty} \frac{3n^2 + 10n + 7}{3n^2 + 10n}$$

$$= 1$$

Thus, the series $\sum_{n=1}^{\infty} \frac{n}{3n+7}$ diverges.

Determine if the shown justification is valid.

- This justification is valid.
- This justification is not valid, because the Root Test is inconclusive when the limit tends to 1.
- This justification is not valid, because the series converges to 1.
- This justification is not valid, because the Ratio Test is inconclusive when the limit tends to 1.
- This justification is not valid, because the Root Test should have been applied instead.

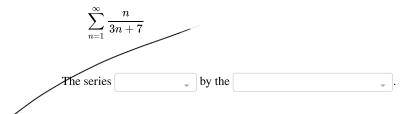
Calculate the limit.

$$\lim_{n\to\infty}\frac{n}{3n+7}$$

(Express numbers in exact form. Use symbolic notation and fractions where needed.)

$$\lim_{n\to\infty}\frac{n}{3n+7}=$$

Use the limit to complete the statement whether the series converges or diverges.



Question 2 of 16

Find $\sqrt[k]{|a_k|}$ for the series.

$$\sum_{k=0}^{\infty} \left(\frac{k}{k+15} \right)^k$$

(Express numbers in exact form. Use symbolic notation and fractions where needed.)

$\sqrt[k]{ a_k } =$	

Use the Root Test to determine the correct stetement.

The Root Test is inconclusive.

The series diverges.

The series converges.

Question 3 of 16

the series
$$\frac{\sum_{n=0}^{\infty} \frac{(n!)^3}{(8n)!}}{\left(\frac{8}{n+1}\right)!} \cdot \frac{\left(\frac{8}{n}\right)!}{\left(\frac{8}{n+1}\right)!} = \frac{\left(\frac{1}{n+1}\right)!}{\left(\frac{8}{n+8}\right)(8n+7)+...1(8n+1)(8n+1)(8n)!} \approx \frac{n^3}{n^3} \Rightarrow 6$$

find the ratio
$$\left| \frac{a_{n+1}}{a_n} \right|$$
.

(Express numbers in exact form. Use symbolic notation and fractions where needed.)

$$\left|\frac{d_{n+1}}{d_n}\right| =$$

Use the Ratio Test to determine the correct statement.

- The series diverges.
- The Ratio Test is inconclusive.
- The series converges.

Question 4 of 16

Given the series

$$\sum_{n=1}^{\infty} \frac{7^n}{n}$$

$$\frac{7^{n+1}}{n+1} \cdot \frac{\eta}{7^n} = \frac{7n}{n+1} \rightarrow$$

find the ratio $\left| \frac{a_{n+1}}{a_n} \right|$.

(Express numbers in exact form. Use symbolic notation and fractions where needed.)

$$\begin{vmatrix} a_{n+1} \\ a_n \end{vmatrix} =$$

Use the Ratio Test to determine the correct statement.

- The series converges.
- The series diverges.
- The Ratio Test is inconclusive.

Question 5 of 16

Use the Root Test to determine convergence or divergence (or state that the test is inconclusive) of the given series.

\sum_{∞}	1
<u>Z</u>	6 ⁿ

Choose the correct answer.

The series converges condition	onally.
--------------------------------	---------

The series diverges.

∞	The	series	converges	absol	lutely	ú

The test is inconclusive.

Question 6 of 16

Apply the Ratio Test to determine convergence or divergence of the given series, or state that the Ratio Test is inconclusive.

$$\sum_{n=1}^{\infty} \frac{5n+2}{7n^3+1} \qquad \text{WDL} \qquad \frac{5n+2}{7n^3+1} \cdot \frac{7n^3}{5n} = 1$$

- The series converges absolutely.
- The series diverges.
- The test is inconclusive.
- The series converges but does not converge absolutely.

Question 7 of 16

Determine convergence or divergence of the series using any method.

$$\sum_{n=1}^{\infty} \frac{n^3}{n!} \qquad \frac{\binom{n+1}{3}}{\binom{n+1}{k}} \qquad \frac{\binom{n+1}{3}}{\binom{n+1}{3}} \qquad = \frac{\binom{n+1}{3}}{\binom{n+1}{3}} \qquad \approx \frac{\binom{n}{3}}{\binom{n+1}{3}} \qquad \approx \frac{\binom{n}{3}}{\binom{n+1}{3}} \qquad \Rightarrow \qquad 0$$

Choose the correct answer.

- The convergence of the series cannot be determined.
- The series diverges.
- The series converges conditionally.
- The series converges absolutely.

Question 8 of 16

Determine	convergence of	r divergence	of the	ceries	neina	anv	method
Determine	convergence c	n divergence	or the	201102	using	any	memou.

$$\sum_{n=1}^{\infty} \frac{n^{11}}{n!}$$

CI.	. 1		
Choose	the	correct	answer.

()	The convergence	of the	series	cannot	be	determined.
	,	The commengemen	01 1110	001100	•	-	acter mine a.

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		diverge	

	The	series	converges	conditionally	v
- 1	1110	SULIUS	Converges	Conditionan	y

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- (.)	1110	201102	converges	ausu.	lutei	ιy

Question 9 of 16

Use the Root Test to determine convergence or divergence (or state that the test is inconclusive) of the given series.

$$\sum_{k=0}^{\infty} \left(\frac{k}{3k+1} \right)^k$$

Choose the correct answer.

- The series diverges.
- The series converges absolutely.
- The test is inconclusive.
- The series converges conditionally.

With $a_k = \left(\frac{k}{2k+1}\right)^k$,

$$\sqrt[k]{a_k} = \sqrt[k]{\left(\frac{k}{2k+1}\right)^k} = \frac{k}{2k+1}$$

Consider the limit of the kth roots at infinity.

$$L = \lim_{k \to \infty} \sqrt[d]{|\overline{a_k}|} = \lim_{k \to \infty} \left(\frac{k}{2k+1}\right) = \frac{1}{2} < 1$$

Therefore, the series $\sum_{k=0}^{\infty} \left(\frac{k}{2k+1}\right)^k$ converges absolutely by the Root Test.

Question 10 of 16

Assume that $\left|\frac{a_{n+1}}{a_n}\right|$ converges to $\rho = \frac{1}{6}$. What can you say about the convergence of the given series?



Choose the correct answer.

- The series converges conditionally.
- The test is inconclusive.
- The series diverges.
- The series converges absolutely.

Use the Ratio Test. Let $b_n = a_n^2$. Then,

$$\rho_b = \lim_{n \to \infty} \left| \frac{b_{n+1}}{b_n} \right| = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|^2 = \left(\frac{1}{6} \right)^2 = \frac{1}{36}$$

Since $\rho_b < 1$, the series $\sum_{n=1}^{\infty} a_n^2$ converges absolutely by the Ratio Test.

Question 11 of 16

Assume that
$$\left|\frac{a_{n+1}}{a_n}\right|$$
 converges to $\rho = \frac{1}{4}$. What can you say about the convergence of the given series?
$$\sum_{n=1}^{\infty} 5^n a_n \qquad \text{set} \quad b_n = 5^n c_n \qquad \left| \quad \frac{b_{n+1}}{b_n} \right| = \frac{5^{n+1} a_{n+1}}{5^n a_n} = 5 \left| \frac{a_{n+1}}{a_n} \right| = \frac{5}{4} > 1 \quad \text{div.}$$

Choose the correct answer.

- The series converges conditionally.
- The series diverges.
- The test is inconclusive.
- The series converges absolutely.

Question 12 of 16

Apply the Ratio Test to determine convergence or divergence of the given series, or state that the Ratio Test is inconclusive.

$$\sum_{n=1}^{\infty} \frac{7^n}{n!} \qquad \frac{7^{n+1}}{(n+1)!} \frac{n!}{7^n} = \frac{7}{n+1} \rightarrow 0 < 1 \implies \text{converge}$$

- The series converges absolutely.
- The test is inconclusive.
- The series converges but does not converge absolutely.
- The series diverges.

Question 13 of 16

The following limit could be helpful to answer the question.

$$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e \qquad \left(\frac{2(n+1)}{n+1}\right)^{\frac{1}{n+1}} - \frac{n}{(2n)!} = \frac{(2n+3)(2n+1)n^n}{(n+1)^{n+1}}$$

$$= \frac{(2n+3)(2n+1)n^n}{(n+1)^n}$$

$$= \frac{(2n+3)(2n+1)n^n}{(n+1)^n} = \frac{n^n}{n^n}$$

$$(2n+1)^n \cdot (n+1) = \frac{n^n}{n^n}$$

$$(2n+1)^n \cdot (n+1) = \frac{2(n+1)}{n^n}$$

$$(2n+1)^n \cdot (n+1) = \frac{2(n+1)}{(n+1)^n}$$

$$(2n+1)^n \cdot (n+1) = \frac{2(n+1)}{(n+1)^n}$$

$$(2n+3)(2n+1) = \frac{2(n+1)}{(n+1)^n}$$

$$(2n+3)(2n+1) = \frac{2(n+3)(2n+1)}{(n+1)^n}$$

$$(2n+3)(2n+1) = \frac{2(2n+1)(2n+1)}{(2n+1)^n}$$

$$(2n+3)(2n+1) = \frac{2(2n+1)(2n+1)}{(2n+1)^n}$$

$$(2n+3)(2n+1) = \frac{2(2n+1)(2$$

Question 14 of 16

The following limit could be helpful to answer the question.

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e\qquad \frac{\left(n+1\right)!}{\left(n+1\right)^{n+1}}\cdot\frac{n^n}{n!}=\frac{\left(n+1\right)n^n}{\left(n+1\right)\left(n+n\right)^n}=\left(\frac{n}{n+1}\right)^n=\left(1+\frac{1}{n}\right)^n\to e$$

Does $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ converge or diverge? The series diverges.

- The series converges absolutely.
- The test is inconclusive.
- The series converges conditionally.

Question 15 of 16

Use the Root Test to determine convergence or divergence (or state that the test is inconclusive) of the given series.

$$\sum_{n=3}^{\infty} \left(1 + \frac{1}{n}\right)^{-n^2} = \sum_{3}^{\infty} \left[\left(1 + \frac{1}{n}\right)^{-n}\right]^n$$

$$\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^{-n} = \left[\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^{n}\right]^{-1} = e^{-1} = \frac{1}{c} < 0$$
Choose the correct answer.

- The test is inconclusive.
- The series diverges.
- The series converges absolutely.
- The series converges conditionally.

Question 16 of 16

Solve for the value of k that makes the series converge.

$$\sum_{n=1}^{\infty} \frac{7^n}{n^k} \quad \frac{7^{n+1}}{(n+1)^K} \cdot \frac{n^K}{7^n} = \frac{7^n}{(n+1)^K} \longrightarrow 7$$

(Use symbolic notation and fractions where needed. If such value does not exist, enter DNE.)

k = DNE