

some basics

A sequence $\{a_n\}_0^{+\infty} = a_0, a_1, a_2, \dots$ **converges** to L if

$$\lim_{n \rightarrow +\infty} a_n = L$$

More formally, a sequence $\{a_n\}_0^{+\infty}$ converges to L if, given an $\epsilon > 0$, there exists an N such that for all $n > N$, $|a_n - L| < \epsilon$.

A sequence $\{a_n\}_0^{+\infty} = a_0, a_1, a_2, \dots$ is **increasing** if
 $a_n \leq a_{n+1}$ for all $n \geq 0$.

A sequence $\{a_n\}_0^{+\infty} = a_0, a_1, a_2, \dots$ is **strictly increasing** if
 $a_n < a_{n+1}$ for all $n \geq 0$.

A sequence $\{a_n\}_0^{+\infty} = a_0, a_1, a_2, \dots$ is **decreasing** if
 $a_n \geq a_{n+1}$ for all $n \geq 0$.

A sequence $\{a_n\}_0^{+\infty} = a_0, a_1, a_2, \dots$ is **strictly decreasing** if
 $a_n > a_{n+1}$ for all $n \geq 0$.

A sequence $\{a_n\}_0^{+\infty} = a_0, a_1, a_2, \dots$ is **monotonic** if
it is either increasing or decreasing.

A **series** is written this way:

$$\sum_{n=0}^{+\infty} a_n, \text{ which means } a_0 + a_1 + a_2 + \cdots$$

a_0, a_1, a_2, \dots is called the **sequence of underlying terms**.

There are two sequences associated with every series:
The sequence $\{s_n\}_0^{+\infty} = s_0, s_1, s_2, \dots$, where

$$s_0 = a_0$$

$$s_1 = a_0 + a_1$$

$$s_2 = a_0 + a_1 + a_2$$

\vdots

$$s_n = a_0 + a_1 + a_2 + \cdots + a_n$$

$$s_{n+1} = a_0 + a_1 + a_2 + \cdots + a_n + a_{n+1}$$

is the **sequence of partial sums**.

A series **converges** if and only if its sequence of partial sums converges.