world of series

1. Determine whether or not the series converges. If the series converges, then find its sum.

(a)

$$\sum_{k=0}^{\infty} 2\left(\frac{2}{3}\right)^k$$

(b)
$$\sum_{k=1}^{\infty} \left(\frac{2}{3}\right)^{k+2}$$

(c)

$$\sum_{k=1}^{\infty} \left(-\frac{3}{2}\right)^k$$

(d)

 $\sum_{k=0}^{\infty} 5^{3k} 7^{1-k}$

2. In each part, find all values of c for which the series converges, and find the sum of the series (the sum will still have a "c" in it). (a)

$$c - c^3 + c^5 - c^7 + c^9 - \cdots$$

(b)
$$e^{-c} + e^{-2c} + e^{-3c} + e^{-4c} + e^{-5c} + \cdots$$

3. Show that

$$\sum_{k=1}^{\infty} \frac{2}{k^2 + 2k} = \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+2}\right) = \frac{3}{2}$$