

February 20, 2025

**Show your work to receive full credit.**

1. In each of the following, determine convergence/divergence. Indicate which test(s) you are using.

(1.1) Indicate absolute convergence, conditional convergence or divergence

$$\sum_{k=0}^{\infty} \frac{(-1)^n}{2n}$$

$$(1.2) \sum_{k=2}^{\infty} \frac{2k\sqrt[3]{k}}{3k^2 + 5k + 1}$$

$$(1.3) \sum_{k=1}^{\infty} \cos\left(\frac{1}{k^2}\right)$$

$$(1.4) \sum_{k=1}^{\infty} \left[ \frac{8}{5} - \frac{\sqrt{k}}{2} \right]^k$$

$$(1.5) \sum_{k=2}^{\infty} \frac{7k}{k^3 + 17}$$

$$(1.6) \sum_{k=0}^{\infty} \frac{5^{3k}}{(2k)!}$$

$$(1.7) \sum_{k=1}^{\infty} \sin\left(\frac{1}{k}\right)$$

2. Prove the following statement: If  $\sum a_n$  converges, then  $\lim_{n \rightarrow +\infty} a_n = 0$

3. Find the value of the convergent series below:

$$(3.1) \sum_{k=1}^{+\infty} \frac{2^{k+1}}{3^{k-1}}$$

$$(3.2) \sum_{k=2}^{+\infty} \left[ 64^{1/k} - 64^{1/(k+2)} \right]$$

4. Give three examples (each) of . . .

(4.1) a divergent alternating series

(4.2) a conditionally convergent alternating series.

(4.3) an absolutely convergent alternating series

(4.4) a decreasing *sequence* that converges to  $\ln 7$ .

(4.5) a strictly increasing *sequence* that converges to  $e$ .

Scratchwork