Sanity Check - Sequences & Series

▾1. What is a sequence?

a. ordered list of numbers

▾2. What is an infinite series?

a. infinite sum : $\sum_{n=1}^{\infty} a_n$

Monotone

$a_n \le a_{n+1} \quad \forall n$

or

$a_n \geq a_{n+1}$

▾3. Give an example of a convergent sequence.

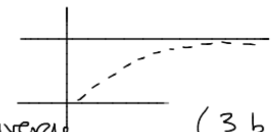
a. 1, 1, 1, 1, 1, 1, 1, 1, 1, 1

▾b. Give an example of a strictly decreasing convergent sequence.

i. $\{\frac{1}{x}\}$, $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \longrightarrow 0$

▾c. Give an example of a strictly increasing convergent sequence.

i. $1 - \frac{1}{x} = \frac{x}{x} - \frac{1}{x} = \frac{x-1}{x}$ $\quad \{1 - \frac{1}{x}\}$



▾4. State the Monotone Convergence Thereom.

a. Sequence + Bounded + Monotone $\Longrightarrow$ Converge

(3 b, c above are such examples)

▾5. Give an example of a divergent sequence.

a. $\{n+1\}_{n=1}^{\infty} = 2, 3, 4, 5, 6, \ldots$ or $\{(-1)^n\}$

▾b. Give an example of a sequence that diverges to infinity.

i.

▾6. Give an example of a convergent series.

a. $\sum 3(\frac{1}{2})^n \longrightarrow$ geometric $-1 < r < 1$ $\Longrightarrow \frac{a}{1-r} \rightarrow \frac{3}{1-\frac{1}{2}} = 6$

▾7. Give an example of a divergent series.

a. $\sum_{n=1}^{\infty} (n+1) = 2 + 3 + 4 + 5 + \ldots$

$a_n = 0$

$\sum_{n=1}^{\infty} a_n = 0 + 0 + 0 \ldots = 0$

▾8. What is a geometric series?

a. ratio of consecutive terms is constant

▾b. When does it converge? Why? Formula proved in class

i.

▾9. What is the harmonic series?

a. $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \ldots$

▾b. When does it converge? Why?

i. Never — It's a slowly divergent series (Bernoulli's Proof)

▾10. How can you tell if a series diverges?

a. $\sum_{n=1}^{\infty} a_n$ will diverge if $\lim_{n \to \infty} a_n \neq 0$

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$$\lim_{n\to\infty} \frac{4}{2^n} + 16\arctan(n^7)$$

$$=$$

$$\lim_{n\to\infty} \frac{4}{2^n} + 16\underbrace{\lim_{n\to\infty}\arctan(n^7)}$$

$$\downarrow$$

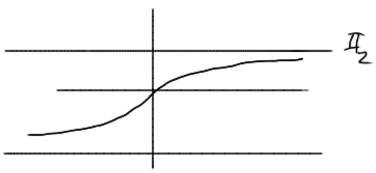
$$0 \qquad 16\cdot\arctan\left(\lim_{n\to\infty} n^7\right)$$

$$=$$

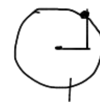
$$16\cdot\underbrace{\arctan(\infty)}_{\frac{\pi}{2}}$$

large + #

what angle gives wicked huge slope



$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x} = $ slope



(x,y)

} rise $= y$

run $= x$

	In	Out
tan:	angle	slope
arctan:	slope	angle

$\uparrow \qquad \uparrow$

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For the series which conve...

(a) $\sum_{n=1}^{\infty} \frac{9^n}{8^n} =$ ______ $\left(\frac{9}{8}\right)^n = \sum_{n=1}^{\infty} ar^n$

(b) $\sum_{n=2}^{\infty} \frac{1}{2^n} =$ ______ $\left(\frac{1}{2}\right)^n$

(c) $\sum_{n=0}^{\infty} \frac{2^n}{6^{2n+1}} =$

(d) $\sum_{n=5}^{\infty} \frac{8^n}{9^n} =$

(e) $\sum_{n=1}^{\infty} \frac{6^n}{6^{n+4}} =$

(f) $\sum_{n=1}^{\infty} \frac{8^n + 2^n}{9^n} =$ $\sum \frac{8^n}{9^n} + \sum \frac{2^n}{9^n}$

$\frac{A+B}{C} = \frac{A}{C} + \frac{B}{C}$

$A^N \cdot A^M = A^{N+M}$

$(A^B)^C = A^{BC}$

$\left(\frac{A}{B}\right)^n = \frac{A^n}{B^n}$

$\frac{2^n}{6^{2n+1}} = \frac{2^n}{6^{2n} \cdot 6^1} = \frac{1}{6} \cdot \frac{2^n}{6^{2n}} = \frac{1}{6} \frac{2^n}{(6^2)^n} = \frac{1}{6}\left[\frac{1}{18}\right]^n$

$\frac{1}{6}\left[\frac{2^n}{36^n}\right] = \frac{1}{6}\left(\frac{2}{36}\right)^n$

$\sum_{n=0+1}^{\infty} \frac{1}{6}\left(\frac{1}{18}\right)^{n-1} = \sum_{n=1}^{\infty} \frac{1}{6}\left(\frac{1}{18}\right)^{n-1}$

$n=0$ $n=1$

$\frac{1}{6}$ $\frac{1}{6 \cdot 18}$