Monday — Week 6

Exam 1

- curved grades on EduCat
- To keep your curved score
  - re-take exam at home.
  - open world.
  - Thursday \ Friday
  - this applies if your curved score < 90
  - if your curved score > 90 do nothing. , still get curved score

sequences, . . . , sequences, . . .

1. Find the general term for each of the following sequences. Then determine whether or not the sequence converges. If the sequence does converge, find its limit.

(a)

$$\frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \frac{1}{256}, \frac{1}{1024}, \ldots \qquad \frac{1}{4^n}$$

$$= \frac{1}{4}, \frac{1}{4^2}, \frac{1}{4^3}, \frac{1}{4^4}, \ldots$$

$$\lim_{n\to\infty} \left(\frac{1}{4}\right)^n = \lim_{n\to\infty} \frac{1}{4^n} = 0$$

L'Hospital's $\left(\frac{\infty}{\infty} \text{ or } \frac{0}{0}\right)$

Fact

$$\underbrace{\lim_{n\to\infty}}_{\text{natural \#'s}} f(n) = \underbrace{\lim_{x\to\infty}}_{\text{real \#'s}} f(x)$$

(b)

$$\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{4}{6}, \frac{5}{7}, \ldots \qquad \frac{n}{n+2}$$

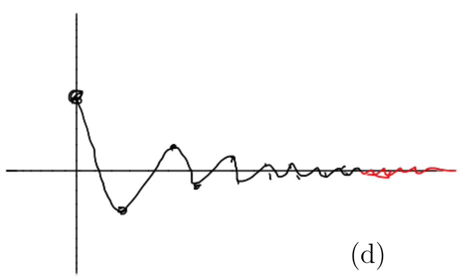
$$\lim_{n\to\infty} \frac{n}{n+2} = \lim_{x\to\infty} \frac{x}{x+2} = \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{1}{1} = 1$$

(c)

$$1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, \ldots \qquad (-1)^n \cdot \left(\frac{1}{2}\right)^n = \left(-\frac{1}{2}\right)^n$$

starting @ $n=0$

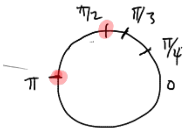


$$\lim_{n\to\infty} \left(\frac{-1}{2}\right)^n = \lim_{n\to\infty} \frac{(-1)^n}{2^n} = 0$$

(d)

$$1, -1, 1, -1, 1, \ldots = (-1)^n$$

divergent



2. Write out the first six terms of the sequence below (decimal form). Does the sequence converge or diverge?

$$\left\{ n \sin\left(\frac{\pi}{n}\right) \right\}_{1}^{+\infty}$$

$\frac{\infty}{\infty}$ $\quad$ $\frac{0}{0}$

$n=1$

$= 1\cdot\sin(\pi),\ 2\cdot\sin\left(\frac{\pi}{2}\right),\ 3\cdot\sin\left(\frac{\pi}{3}\right),\ 4\cdot\sin\left(\frac{\pi}{4}\right),$

$= 0,\ 2,\ 3\cdot\frac{\sqrt{3}}{2},\ 4\cdot\frac{\sqrt{2}}{2},\ \ldots$

$\left(\frac{\pi}{n}\right)' = \pi n^{-1}$

$= -\frac{\pi}{n^2}$

$= 0, 2, 2.598, 2.828, 2.939, \ldots$

$$\lim_{n\to\infty} n\cdot\sin\left(\frac{\pi}{n}\right) = \lim_{n\to\infty} \frac{\sin\left(\frac{\pi}{n}\right)}{\frac{1}{n}} = \frac{0}{0}$$

$\infty\cdot\sin\left(\frac{\pi}{\infty}\right)$

$\boxed{\infty\cdot 0} = ?$

$$\overset{L'H}{=} \lim_{n\to\infty} \frac{-\frac{\pi}{n^2}\cos\left(\frac{\pi}{n}\right)}{-\frac{1}{n^2}} = \lim_{n\to\infty} \pi\cdot\cos\frac{\pi}{n}$$

$\cos 0 = 1$

$= \boxed{\pi}$

3. Write the first five terms of the sequence defined below (decimal form). If the sequence converges, what is its limit?

$n=1 \quad n=2$

$a_{n+1} = \sqrt{a_n + 2}, \quad a_1 = 0$ $\qquad$ Recursive Seq.

$\{a_n\} = 0,\ \sqrt{2},\ \sqrt{2+\sqrt{2}},\ \sqrt{\sqrt{2+\sqrt{2}}+2},\ \sqrt{\sqrt{\sqrt{2+\sqrt{2}}+2}+2}, \ldots$

Key: limit of a sequence does not depend on finitely many terms.

$$\lim_{n\to\infty} a_n \overset{*}{=} \lim_{n\to\infty} a_{n+1}$$

Set $L = \lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1} \overset{\text{sub}}{=} \lim_{n\to\infty} \sqrt{a_n + 2}$

$$= \sqrt{\lim_{n\to\infty} a_n + 2}$$

$L = \sqrt{L+2}$ $\qquad$ the limit $= 2$

$L^2 = L + 2 \Rightarrow L^2 - L - 2 = 0$

$(L-2)(L+1) = 0$ $\qquad$ $\boxed{L = 2}$ $\quad$ $\cancel{L = -1}$