MA143 Wk 6 - Mon

Exam 2 (ch. 10 - 10.5)
next week

(10.3) Limit Comparison Test

For $\{a_n\}$, $\{b_n\}$ positive sequences and $L = \lim_{n\to\infty} \frac{a_n}{b_n}$

$L > 0$: $L = \frac{\lim a_n}{\lim b_n}$ cross mult $L \cdot \lim b_n = \lim a_n$

limit exists and is positive

since this is a positive # $\Rightarrow$ two sequences grow @ roughly the same rate

a_n converges $\Leftrightarrow$ b_n converges

$L = \infty$: $\boxed{\infty = \frac{\lim a_n}{\lim b_n}}$ $\Rightarrow$ $\infty \cdot \lim b_n = \lim a_n$

The denom is growing to infinity SLOWER than the convergent series a_n, thus the b_n series converges

If $\sum a_n$ converges (thus $\lim a_n = 0$) so $\infty \cdot \underbrace{\lim b_n}_{=0} = 0$

then $\sum b_n$ converges

$L = 0$: $0 = \frac{\lim a_n}{\lim b_n}$ $\Rightarrow$ $0 \cdot \lim b_n = \lim a_n$

Here the b_n are growing MUCH faster then the a_n, so if the b-series converges the a-series converges

Ex $\sum_{n=1}^{\infty} \frac{n^2}{n^4 - n + 1}$

Does this series converge?

think: see fraction, think about the terms as n grows, in which case … only the leading terms will matter

$\approx \frac{n^2}{n^4} = \boxed{\frac{1}{n^2}}$ compare given series to this

You should know $\sum \frac{1}{n^2}$ converge

p-series $p = 2 > 1$

$\frac{1}{n} \rightarrow$ div $p = 1$

$\frac{\frac{A}{B}}{\frac{C}{D}} = \frac{A}{B} \cdot \frac{D}{C}$

compare $\lim_{n\to\infty} \frac{\frac{1}{n^2}}{\frac{n^2}{n^4 - n + 1}} \cdot \lim_{n\to\infty} \frac{1}{n^2} \cdot \frac{n^4 - n + 1}{n^2} = \lim_{n\to\infty} \frac{n^4 - n + 1}{n^4} = \frac{\infty^4 - \infty + 1}{\infty^4} = \frac{\infty}{\infty}$

(L'H $\lim \frac{4n^3 - 1}{4n^3} = \frac{\infty}{\infty}$

(L'H $\lim \frac{12n^2}{12n^2} = \lim_{n\to\infty} 1 = 1$

By L.C.T., since $\sum \frac{1}{n^2}$ converges then our series converges

Ex $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2+5}}$

find a sequence to compare it to: $\frac{1}{n}$

$\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{\sqrt{n^2+5}}} = \lim_{n\to\infty} \frac{\sqrt{n^2+5}}{n} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}$ $\frac{1}{n} = \frac{1}{\sqrt{n^2}}$ $= \lim \frac{\frac{\sqrt{n^2+5}}{\sqrt{n^2}}}{\frac{n}{n}}$

$= \lim_{n\to\infty} \frac{\sqrt{1 + \frac{5}{n^2}}}{1} = \frac{\sqrt{1+0}}{1} = 1$

$4 = \sqrt{4^2}$

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$\sum \frac{1}{n}$ diverges $\Rightarrow$ our series diverges

Harmonic

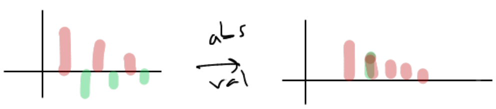
How to deal with series that are not always positive

Def: absolute convergence
The series $\sum a_n$ converge absolutely if $\sum |a_n|$ converges.

Ex. $1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \frac{1}{36} + \dots = \sum_{n=1} \frac{(-1)^{n+1}}{n^2}$ Does it converge absolutely?

$\sum \left| \frac{(-1)^{n+1}}{n^2} \right| = \sum \frac{1}{n^2}$ converges (p-test)

For "alternating series" — it's "harder" for a series to converge absolutely.



$\boxed{\text{theorem: Absolute Convergence implies Convergence}}$

Ex. Does $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4}$ converge?

1. Abs val: $\sum_{n=1}^{\infty} \left| \frac{(-1)^{n+1}}{n^4} \right| = \sum_{n=1}^{\infty} \frac{1}{n^4}$ converges $p > 1$
2. this series conv. absolutely, thus it converges.

Ex $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ Does it converge?

1. abs val $\Rightarrow$ $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverge
2. careful ... don't apply Abs Conv. Test.

We'll see: ALT. SERIES TEST

If $\{b_n\}$ is positive & decreasing $\Rightarrow \sum (-1)^n b_n$ converges

($\frac{1}{\sqrt{n}}$ is decreasing, & By A.S.T. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ converges.