Thursday - Week 6

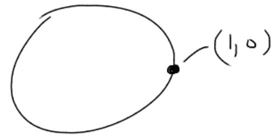
warm-up:

$$\lim_{n\to\infty} \cos(1/n^2) = 1$$

$$\| $$

$$\cos\left(\lim_{n\to\infty}(1/n^2)\right) = \cos(0)$$

x-coord @ radian 0



- Exam - Redo's
  Today / Tomorrow
- WeBwork 1

Tests for convergence / divergence

Often we only care about whether a series converges or not.

Divergence Test:

If a series converges then the terms get infinitely small. $\left(\lim_{n\to\infty} a_n = 0\right)$

First recall $S_N = a_1 + a_2 + a_3 + \dots + a_N$ (the N^{th} partial sum)

$$\boxed{\lim_{N\to\infty} S_N = \sum_{n=1}^{\infty} a_n}$$

Assume $\sum_{n=1}^{\infty} a_n$ converges $\left(\sum_{n=1}^{\infty} a_n = S\right)$

fact: $a_{n+1} = S_{n+1} - S_n$ (eg, $a_5 = a_1 + a_2 + a_3 + a_4 + a_5 - (a_1 + a_2 + a_3 + a_4)$)

or

$$\boxed{a_n = S_n - S_{n-1}}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}(S_n - S_{n-1}) = \lim_{n\to\infty} S_n - \lim_{n\to\infty} S_{n-1} = S - S = 0$$

If series conv then limit $= 0$ (if P then Q)

$$\boxed{\text{If } \lim_{n\to\infty} a_n \neq 0 \text{ then the series diverges.}}$$

if not Q then not P

Examples: Apply Divergence Test to determine if they diverge.

$\sum_{n=1}^{\infty} \frac{n}{3n-1}$ | $\lim_{n\to\infty} \frac{n}{3n-1} = \frac{1}{3} \neq 0$ By Div. Test series diverges.

$\boxed{\text{If } \lim_{n\to\infty} a_n \neq 0 \text{ then the series diverges.}}$

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ | $\lim_{n\to\infty} \frac{1}{n^2} = 0$ | The divergence test is inconclusive.

$\sum_{n=1}^{\infty} e^{1/n^2}$ | $\lim_{n\to\infty} e^{1/n^2} = 1$

Apply Div. Test $\Rightarrow$ Diverges

Sufficient | Neccessary

$\frac{1}{n^2} \longrightarrow 0$ and $\sum \frac{1}{n^2}$ converges

$\frac{1}{n} \longrightarrow 0$ yet $\sum \frac{1}{n}$ diverges

terms approaching zero is Neccessary for series to converge

Fact:

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \dots = \frac{\pi^2}{6}$$

proved by

e $\longleftrightarrow$ Euler

Quickly

$\sum_{n=1}^{\infty} \cos(1/n^2) =$ diverges b/c $\lim_{n\to\infty} \cos(1/n^2) = 1$ (div test)