MA163 wk 6 thur

Caution: Don't conflate sequence & series

$\frac{1}{n^2}$

series: $\sum_{n=1}^{\infty} \frac{1}{n} = \infty$ (diverges to ∞), $\sum_{n=1}^{\infty} \frac{1}{n^2} = \lim_{K\to\infty} S_K$ → K^{th} partial sum

sum

$\frac{1}{1}$

$\frac{1}{1}+\frac{1}{4}$

$\frac{1}{1}+\frac{1}{4}+\frac{1}{9}$

$\frac{1}{1}+\frac{1}{4}+\frac{1}{9}+\dots+\frac{1}{K^2}$ ← S_K

sequence: $\{\frac{1}{n}\}$ or $(\frac{1}{n})$ or even better $\{\frac{1}{n}\}_{n=1}^{\infty}$

list

$\lim_{n\to\infty} \frac{1}{n} = 0$

Q: How to we find/evaluate what a sequence converges to?

- In a few cases we have formulas:

1. geometric series: $\sum_{n=0}^{\infty} cr^n = \frac{c}{1-r}$ when $|r|<1$

2. Find a pattern (formula) for S_K, take limit

$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

$S_1 = \frac{1}{2}$

$S_2 = \frac{1}{2} + \frac{1}{6}$

$S_3 = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \dots$

or telescoping series $\underbrace{\sum_{n=1} \frac{1}{n} - \frac{1}{n+1}}$ easier to get formula for S_K

$\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$

$1 = A(n+1) + Bn$

$\underbrace{A+B=0}$, $A=1$

no n term on r.h.t

$S_1 = \frac{1}{1} - \frac{1}{2} = \frac{1}{2}$

$S_2 = \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} = 1 - \frac{1}{3}$

$S_3 = 1 - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} = 1 - \frac{1}{4}$

$S_4 = \underbrace{1 - \frac{1}{4}}_{S_3} + \underbrace{\frac{1}{4} - \frac{1}{5}}_{a_4} = 1 - \frac{1}{5}$

$S_K = 1 - \frac{1}{K+1}$

ans $\lim_{K\to\infty} S_K = 1$

3. Often, this hard, it's enough to know if div, forget about if conv, estimate.

10.4

Homework exercises:

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \cdot \ln(n^3)}$$

- ☐ converge absolutely (take abs value of terms, still converges) → converge b/c terms get small fast
- ☐ converge conditionally (the alt. series converges, but the abs. value version doesn't.
- ☐ diverge

$\Rightarrow \sum \frac{(-1)^n}{n}$ converges conditionally

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ conv – b/c A.S.T.

ALT. HARM

① ALT

② decrease $\frac{1}{n}$ (deriv < 0)

$\sum_{n=1}^{\infty} \frac{1}{n}$ div

HARM

① A.S.T. $\left\{ \frac{1}{n \cdot \ln(n^3)} \right\}$ is decreasing b/c numerator is const, & denom is increasing:

$n \cdot \ln(n^3) = 3 \cdot \ln(n) \cdot n$

$\frac{d}{dn}$

$3\left(\frac{1}{n} \cdot n + \ln(n)\right) = 3(1 + \ln(n))$

For $n > 2$ this is $+$

By A.S.T., series converges

$3A + 3B = 3(A+B)$

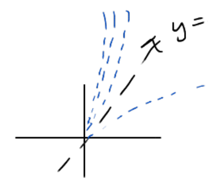
look @ pos. series

② $\sum_{n=2} \frac{1}{n \cdot \ln(n^3)} = \sum \frac{1}{n \cdot 3\ln(n)}$

$= \frac{1}{3} \sum_{n=2}^{\infty} \frac{1}{n \cdot \ln(n)}$

Try to compare:

$\frac{1}{n \cdot \ln(n)} < \frac{1}{n}$ div $\Rightarrow$ unhelpful



$\frac{1}{n \cdot \ln(n)} \overset{?}{<} \frac{1}{n^2}$ (=) $n^2 \overset{?}{<} n \cdot \ln(n)$ false

Since $\sum_{n=2}^{\infty} \frac{1}{n^2}$ converges

Comparison Test doesn't apply

$$\boxed{\text{By Integral Test Series Diverges} \Rightarrow \text{main converges conditionally}}$$

when in doubt, try Integral Test:

$\frac{1}{3} \int_2^{\infty} \frac{1}{x \cdot \ln(x)} dx$ $\quad u = \ln x$, $du = \frac{1}{x} dx$

$= \frac{1}{3} \int_{\ln(2)}^{\infty} \frac{1}{u} du = \frac{1}{3} \ln(|\ln(x)|) \Big|_2^{\infty} = \infty$

Ex

$$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^{4/5}}\right)$$

L.C.T. w/ $\frac{1}{n^{4/5}}$

$\sum_{n=1}^{\infty} \frac{1}{n^{4/5}}$ diverges by test

$$\lim_{n\to\infty} \frac{\sin\left(\frac{1}{n^{4/5}}\right)}{\left(\frac{1}{n^{4/5}}\right)}$$

set $u = \frac{1}{n^{4/5}}$

if $n \to \infty$

$u \to 0$

$$= \lim_{u\to 0} \frac{\sin(u)}{u} = 1$$

$\Rightarrow$ top ~ bottom

$\updownarrow$

series div

$\Downarrow$

top series diverges