Thur wk 6 —

warm-up: Find the flaw in reasoning:

$$\underbrace{\sum_{n=1}^{\infty} \frac{(n+1)^n}{(n^3+1)^n}}_{\text{series}} \sim \underbrace{\left(\frac{n+1}{n^3+1}\right)^n \sim \frac{1}{n^2} \rightarrow \text{converges to } 0}_{\text{sequence}} \text{ so } \underline{\underline{\text{sum}}} \text{ is } 0.$$

the sequence $\frac{1}{n^2} \rightarrow 0$

series

Flaw mixing up: series vs. sequence:

Ex.

$\frac{1}{n}$ → sequence $\left\{\frac{1}{n}\right\} \rightarrow 0$ (converges) | sequence needs $\{\ \}$ braces

$\frac{1}{n}$ → series $\sum_{n=1}^{\infty} \frac{1}{n} \rightarrow$ diverges to ∞ series needs $+$ or Σ

Root Test: $\lim_{n\to\infty} \sqrt[n]{\frac{(n+1)^n}{(n^3+1)^n}} = \lim_{n\to\infty} \frac{n+1}{n^3+1} = 0 \Rightarrow$ converges. since $0<1$

— n are whole #'s

$\lim_{x\to\infty} \frac{x+1}{x^3+1} = \frac{\infty}{\infty} \overset{L'H}{=} \lim_{x\to\infty} \frac{1}{3x^2} = \frac{1}{3\cdot\infty^2} = \frac{1}{\infty} = 0$

real

Some Homework exercises: (10.4)

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \cdot \ln(n^3)}$$

* converges b/c n-th term growth is slow

- ☐ converge absolutely (take the abs. value of terms, new series converges)
- ☐ converge conditionally (alt. series converges, but the positive one diverges
  * converges only b/c of +/− cancellation
- ☐ diverge

① see: alternating, apply A.S.T. $\left\{ \begin{array}{l} \text{if alternating and (+ part) decreasing} \\ \text{then converge} \end{array} \right.$

$\left\{\frac{1}{n \cdot \ln(n^3)}\right\}$ is decreasing for all $n > 2$ b/c it's derivative is negative.

$$\frac{1}{x \cdot \ln(x^3)} = \frac{1}{x \cdot 3 \cdot \ln x} = \frac{1}{3} \frac{1}{x \cdot \ln x} \xrightarrow{d/dx} \frac{-(\ln x + 1)}{(x \cdot \ln x)^2} < 0 \quad \forall x > 2$$

∴ By A.S.T. the given series converges

Ex $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges conditionally

vs

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges

Harmonic

② Does it converge absolutely? $\sum_{n=2}^{\infty} \frac{1}{n \cdot \ln(n^3)}$ does this converge?

$$\frac{1}{n \cdot \ln(n^3)} < \frac{1}{n} \text{ — div.}$$

⇒ comparison test is unhelpful

⇒ use Integral Test (when other other tests fail)

$$\int_2^{\infty} \frac{1}{x \cdot 3 \cdot \ln(x)} dx = \frac{1}{3} \int_2^{\infty} \frac{1}{x \cdot \ln x} dx \qquad u = \ln x \qquad du = \frac{1}{x} dx$$

$$= \frac{1}{3} \int_{\ln 2}^{\infty} \frac{1}{u} du = \frac{1}{3} \ln|u| \Big|_{\ln 2}^{\infty} = \frac{1}{3}\left(\ln(\infty) - \underbrace{\ln(\ln 2)}_{\text{finite}}\right) = \infty$$

By Integral Test, the positive series diverges

③ Since alt. series converged, it's + version didn't ⇒ series converges conditionally

Ex

$$= \sum_{n=2}^{\infty} \ln\left(\frac{n+1}{n}\right) = \sum_{n=2}^{\infty} \ln(n+1) - \ln(n)$$

give k^{th} partial sum ; S_k

S_2: $\ln(3) - \ln(2)$

S_3: $\ln(4) - \ln(3) + \ln(3) - \ln(2)$

S_4: $\ln(5) - \ln(4) + \ln(4) - \ln(3) + \ln(3) - \ln(2)$

$\vdots$

$$S_k = \ln(k+1) - \ln(2) = \ln\left(\frac{k+1}{2}\right)$$

what does S_k have to do with the convergence of the series?

$\infty \Rightarrow$ div.

$c \Rightarrow$ conv.

$$\lim_{k\to\infty} S_k = \sum_{n=2}^{\infty} \ln\left(1+\frac{1}{n}\right)$$

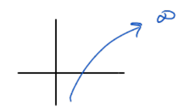
S_2: $\ln\left(1+\frac{1}{2}\right)$

S_3: $S_2 + \ln\left(1+\frac{1}{3}\right)$

$= \ln\left(1+\frac{1}{2}\right) + \ln\left(1+\frac{1}{3}\right)$

$S_4 = \ln\left(1+\frac{1}{2}\right) + \ln\left(1+\frac{1}{3}\right) + \ln\left(1+\frac{1}{4}\right)$

$\vdots \quad = \ln\left(\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\right)$



$$\lim_{k\to\infty} \ln\left(\frac{k+1}{2}\right) = \infty$$

$\Rightarrow$ series diverges

10.4.9

$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^{4/5}}\right)$ $\boxed{}$ same ans. $\boxed{}$

L.C.T. to compare w/ $\frac{1}{n^{4/5}}$. we know $\sum_{n=1}^{\infty} \frac{1}{n^{4/5}}$

$$\lim_{n\to\infty} \frac{\sin\left(\frac{1}{n^{4/5}}\right)}{\frac{1}{n^{4/5}}} =$$

set $u = \frac{1}{n^{4/5}}$

as $n \to \infty$

$u \to 0$

$= \lim_{u\to 0} \frac{\sin(u)}{u} = 1 \Rightarrow$ series are essentially same

$\Rightarrow$