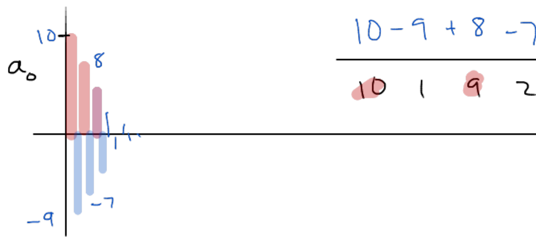
MA 163 - Wk 6 Wed

① Alternating Series Test

② Ratio test

③ Root test

(I) A.S.T. $\sum_{n=1}^{\infty} \underbrace{(-1)^{n-1}}_{\text{sign alternates}} a_n$ if decreasing (i.e., $a_{n+1} < a_n$) implies series converge



$$\frac{10-9+8-7+6-5+4-3+2-1 < 10}{10 \quad 1 \quad 9 \quad 2 \quad 8 \quad 3 \quad 7 \quad 4 \quad 6 \quad 5}$$

↑ current sum

(★) $|S - S_N| < a_{N+1}$

error in approximating the true valuve by the approximated one is less than the first omitted value.

ex $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} = \sum (-1)^n \frac{1}{n}$ alternating Harmonic Series ($\sum \frac{1}{n} \to \infty$ normal Harmonic)

Since: $\frac{1}{n}$ is decreasing

$\frac{1}{n+1} < \frac{1}{n}$

bigger denominator
=
smaller quotient

Apply A.S.T. $\Rightarrow$ $\sum \frac{(-1)^n}{n}$ converges

By (★) $S = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$, $S_{10} = \sum_{n=1}^{10} \frac{(-1)^n}{n} = -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \frac{1}{7} + \frac{1}{8} - \frac{1}{9} + \frac{1}{10}$

$\approx -.65$

But $|S - S_{10}| < \frac{1}{11}$

Question: How far do we have to go out to be within 1/1000 of the true value | Find N s.t.

$|S - S_N| < \frac{1}{1000}$ By (★) upper bound $= \frac{1}{N+1}$

$\frac{1}{1000} = \frac{1}{N+1} \Rightarrow N+1 = 1000$ $N = 999$

Ratio Test

$$\sum_{n=1}^{\infty} a_n$$

[may be positive or negative!]

Root Test

DIVERGENCE TEST

if $a_n \not\to 0$ as $n\to\infty$ $\Rightarrow$ diverge

if series converge, seq. $\{a_n\} \to 0$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \begin{cases} >1 & \text{series diverges} \\ <1 & \text{series converge} \\ =1 & ? \text{ inconclusive} \end{cases}$$

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \begin{cases} >1 & \text{series diverges} \\ <1 & \text{series converge} \\ =1 & ? \text{ inconclusive} \end{cases}$$

Ex $\sum_{n=1}^{\infty} \frac{e^n}{n}$

Ratio test: $\lim_{n\to\infty} \dfrac{\frac{e^{n+1}}{n+1}}{\frac{e^n}{n}}$

$$= \lim_{n\to\infty} \frac{\overset{e^n \cdot e^1}{\overset{\shortparallel}{e^{n+1}}}}{n+1} \cdot \frac{n}{e^n} = \lim_{n\to\infty} \frac{e\cdot n}{n+1} \quad \frac{e\cdot \cancel{n}}{\cancel{n}} = e > 1$$

$\Rightarrow$ series diverges

Ex $\sum_{n=1}^{\infty} \left(\frac{(n+1)^n}{(n^2+1)^n}\right)$ see powers, think root test

$$\hookrightarrow \lim_{n\to\infty} \sqrt[n]{\frac{(n+1)^n}{(n^2+1)^n}} = \lim_{n\to\infty} \frac{n+1}{n^2+1} = 0$$

$\Rightarrow$ series converges

Ex $$\sum_{n=1}^{\infty} \frac{2^n}{n!} = \frac{2}{1} + \frac{2^2}{2!} + \frac{2^3}{3!} + \frac{16}{24} + \frac{32}{\underset{\overset{\shortparallel}{120}}{5!}} + \dots$$

$5! = 5\cdot4\cdot3\cdot2\cdot1$

see: factorial do ratio

$$\boxed{\sum \tfrac{1}{n} = \infty} \qquad \boxed{\lim_{n\to\infty} \tfrac{1}{n} = 0}$$

series — sequence

$\approx \frac{2}{n} \leftrightarrow$ Harmonic series $\rightarrow$ diverge — ∞

$$\lim_{n\to\infty} \frac{\frac{2^{n+1}}{(n+1)!}}{\frac{2^n}{n!}} = \lim \frac{\overset{\overbrace{2\cdot2\cdot2\cdots2}^{n+1}}{\overset{\shortparallel}{2^{n+1}}}}{(n+1)!} \cdot \frac{n!}{2^n} = \lim_{n\to\infty} \frac{2}{n+1} = 0 < 1 \Rightarrow \text{Converges}$$

$2^n \cdot 2$

sequence

$$\frac{5!}{6!} = \frac{\cancel{5\cdot4\cdot3\cdot2\cdot1}}{6\cdot\cancel{5\cdot4\cdot3\cdot2\cdot1}} = \frac{\cancel{5!}}{6\cdot\cancel{5!}}$$