**initial tests . . . .**

1. Use the $p$-series test to determine whether or not the series converges.

$$\sum_{k=1}^{\infty} k^{-4/3} \qquad \sum_{k=1}^{\infty} \frac{1}{\sqrt[4]{k}} \qquad \sum_{k=1}^{\infty} \frac{1}{k^{\pi}}$$

| | | | |
|---|---|---|---|
| $p =??$ | $4/3$ | $1/4$ | $\pi$ |
| conv/div? | conv | div | conv |

2. What does the test for divergence tell you (or not tell you) about these series?

(a) Divergence Test tells us NOTHING.

$$\sum_{k=0}^{\infty} \frac{k}{e^k} \qquad \lim_{x\to+\infty} \frac{x}{e^x} = \lim_{x\to+\infty} \frac{1}{e^x} = 0$$

(b) Divergence Test tells us that the series below DIVERGES.

$$\sum_{k=0}^{\infty} \frac{\sqrt{k}}{\sqrt{k}+3} \qquad \lim_{x\to+\infty} \frac{\sqrt{x}}{\sqrt{x}+3} = \lim_{x\to+\infty} \frac{\frac{1}{2}x^{-1/2}}{\frac{1}{2}x^{-1/2}} = 1$$

3. Find the value of the series below.

$$\sum_{k=2}^{\infty}\left[\frac{1}{k^2-1}-\frac{7}{10^k}\right]=\sum_{k=2}^{\infty}\left[\frac{1/2}{k-1}-\frac{1/2}{k+1}-\frac{7}{10^k}\right]$$

$$=\sum_{k=2}^{\infty}\left[\frac{1}{2k-2}-\frac{1}{2k+2}-\frac{7}{10^k}\right]$$

Underlying sequence of terms and sequence of partial sums:

$$\begin{array}{lclcl}
a_2 & = & \frac{1}{2}-\frac{1}{6}-\frac{7}{100} & \Longrightarrow & s_2 = \frac{1}{2}-\frac{1}{6}-\frac{7}{100} \\
a_3 & = & \frac{1}{4}-\frac{1}{8}-\frac{7}{1000} & \Longrightarrow & s_2 = \frac{1}{2}+\frac{1}{4}-\frac{1}{6}-\frac{1}{8}-\frac{77}{1000} \\
a_4 & = & \frac{1}{6}-\frac{1}{10}-\frac{7}{10000} & \Longrightarrow & s_4 = \frac{1}{2}+\frac{1}{4}-\frac{1}{8}-\frac{1}{10}-\frac{777}{10000} \\
a_5 & = & \frac{1}{8}-\frac{1}{12}-\frac{7}{100000} & \Longrightarrow & s_5 = \frac{1}{2}+\frac{1}{4}-\frac{1}{10}-\frac{1}{12}-\frac{7777}{100000} \\
 & & & \vdots &
\end{array}$$

$$s_n=\frac{1}{2}+\frac{1}{4}-\frac{1}{2n}-\frac{1}{2n+2}-\frac{77\cdots 7}{1000\cdots 0}$$

So . . .

$$\sum_{k=2}^{\infty}\left[\frac{1}{k^2-1}-\frac{7}{10^k}\right]=\lim_{n\to+\infty}\frac{1}{2}+\frac{1}{4}-\frac{1}{2n}-\frac{1}{2n+2}-\frac{77\cdots 7}{1000\cdots 0}$$

$$=\frac{1}{2}+\frac{1}{4}-0.0\overline{7}=\frac{1}{2}+\frac{1}{4}-\frac{7}{90}=\frac{121}{180}$$

4. Use the integral test to determine whether or not the series below converges.

$$\sum_{k=0}^{\infty}\frac{1}{(4+2k)^{3/2}}$$

$$\int_0^{+\infty}\frac{1}{(4+2x)^{3/2}}\,dx==\lim_{b\to+\infty}\int_0^b\frac{1}{(4+2x)^{3/2}}\,dx$$

$$=\lim_{b\to+\infty}\left[\frac{-1}{\sqrt{4+2x}}\bigg|_0^b\right]=\lim_{b\to+\infty}\left(\frac{-1}{\sqrt{4+2b}}-\frac{-1}{\sqrt{4}}\right)=\frac{1}{2}$$

The improper integral converges (and the function $\frac{1}{(4+2x)^{3/2}}$ satisfies the conditions required by the Integral Test). Therefore the infinite sum converges.